Estimating Uncertainties in Measurements of Atmospheric Properties by Radio Occultations

Paul Withers

Center for Space Physics,
Boston University
(withers@bu.edu)

MEX/VEX Radio Science Meeting
2009.04.16-17
Cologne, Germany

Background

- I wanted to predict uncertainties in the atmospheric density and ionospheric electron density that would be measured by a hypothetical radio occultation experiment
- I didn’t see obvious and useful relationships in the literature, so I started to derive some
- Closer reading of the literature suggests that most of what I found has already been published, but in many scattered places

A big, bad assumption

$$s(t) = A(t) \sin [2\pi f(t)t] + \epsilon(t),$$

Lipa and Tyler (1979)

- I neglect noise and consider only errors due to variations in frequency due to oscillator’s Allan Deviation
- This is not a good assumption, but it gives me a place to start
- Next steps are to incorporate noise

Bending angle and refractive index

$$\alpha'(a') = -2a' \int_{r=r'}^{r=\infty} \frac{d \ln \mu(r)}{dr} \frac{dr}{\sqrt{(\mu(r)r)^2 - a'^2}}$$

$\nu = \mu - 1$

$$\nu = \nu_0 \exp\left(\frac{-(r - r')}{H}\right)$$

Assume exponential function to make algebra easy

where H is a scale height. If $|\nu_0| \ll 1$, then:

$$\frac{d \ln \mu}{dr} = \frac{-\nu_0}{H} \exp\left(\frac{-(r - r')}{H}\right)$$

Smallest detectable refractivity

$$\alpha'(a') = \nu_0 \sqrt{\frac{2\pi a'}{H}} = \nu(r')\sqrt{\frac{2\pi a'}{H}}$$

$$AD = \delta f / f$$

Approximate relationship between frequency shift and bending angle is: df / f = v α / c

Minimum detectable frequency shift set by Allan Deviation (Neglects effects of noise)

$$\nu_{min} = \frac{ADc}{v}\sqrt{\frac{H}{2\pi R}}$$

Expression for smallest detectable refractivity which is an estimate of measurement uncertainty

Ionosphere

$$\mu_e^2 = 1 - \frac{N e^2}{4\pi^2 m_e \epsilon_0 f^2}$$

Ionospheric refractive index

$$N_{min} = 4\pi \cdot A D f^2 \cdot \frac{c}{v} \cdot \frac{m_e \epsilon_0}{e^2} \sqrt{\frac{2\pi H}{R}}$$

Minimum detectable electron density

N_{min} proportional to: $(AD / v) \times (H/R)^{0.5}$

N_{min} proportional to: f^2

Neutral Atmosphere

$\nu = \kappa n$

Refractive volume κ (~1E-29 m^3) controls refractivity

$$n_{min} = \frac{AD}{\kappa} \cdot \frac{c}{v} \sqrt{\frac{H}{2\pi R}}$$

Minimum detectable neutral number density

n_{min} proportional to: $(AD / v) \times (H/R)^{0.5}$

n_{min} proportional to: $1/\kappa$

H/R ~ 0.02 (Jupiter) to 0.13 (Pluto)

Examples

Table 4
Parameters for radio occultations at Mars, Jupiter and Titan

	Mars	Jupiter	Titan
AD	[2] 3×10^{-13}	[6] 4×10^{-12}	[11] 2×10^{-13}
f (GHz)	[2] 8.4	[7] 2.3	[11] 8.4
v (km s^{-1})	[3] 3.4	[8] 14	[12] 5.6
H (km)	[4] 10	[4] 25	[13] 20/80
R (km)	[4] 3400	[4] 70,000	[4] 2575
κ (m^3)	[5] 1.8×10^{-29}	[9] 6.2×10^{-30}	[14] 1.1×10^{-29}
N_{min} (m^{-3})	1.0×10^{9}	8.5×10^{7}	[15] 1.3×10^{9}
n_{min} (m^{-3})	3.2×10^{19}	1.0×10^{20}	[16] 3.4×10^{19}
[1] α_{surf} (rad)	1.8×10^{-4}	[10] 0.036	[16] 0.036

Next Steps

- Include effects of noise, which probably dominate in real experiments
- Please give me comments, feedback and suggestions
- Does what I'm looking for already exist in the literature?