THE STABILITY OF WAVE-LIKE PULSAR WINDS

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Abstract

Pulsars are heavy, compact, rapidly rotating stars, surrounded by intense electric and magnetic fields. Plasma near the pulsar surface is accelerated away from the pulsar by the fields to form a *pulsar wind*. The accepted, steady-state pulsar wind model does not correctly predict the observed energy transport in the wind. A wave-like model has been proposed which solves this problem. This model is based on intense, relativistic plasma waves, which are potentially unstable: catastrophic energy loss due to radiation from the charges that make up the wind threatens to destroy the wave. Here we show that the energy losses are negligible provided the plasma is streaming relativistically, as is the case in a pulsar wind; hence the wave-like model is stable.

1 Pulsars and their Outflows

In the year 1054, Chinese astronomers recorded the arrival of a new star in the constellation of Taurus. They described it as being a few inches across and as brilliant as the full Moon. It was visible during the day for a month, and remained visible in the evening sky for a whole year. Then it faded out of human sight until the invention of the telescope.

The Chinese had witnessed the birth of the Crab Nebula. A massive star at the end of its life was destroyed in a cataclysmic explosion, called a supernova. The Crab Nebula that we see today is the remains of the outer layers of the star, ejected by the explosion.

The core of the star was compressed by the explosion until it became one of the densest objects in the Universe. The core is heavier than our Sun, yet measures only ten miles across. A teaspoonful of the core weighs about a billion tonnes.

Before the explosion, the core was rotating slowly. Like a pirouetting iceskater, who rotates faster by drawing in her arms, compression increased the rotation speed of the core to about thirty revolutions per second. The magnetic field around the star also increased in strength, from $\sim 1\mathrm{G}$ to $\sim 10^{12}\mathrm{G}$, as a result of the compression, conserving magnetic flux. We call this compact, rapidly rotating, heavily magnetised object a *pulsar* (Shapiro and Teukolsky, 1983)

Like a giant electric motor, this rotating magnet generates intense, oscillating electromagnetic fields in the space around it. The intense fields create electrons and their antiparticles, positrons, close to the pulsar surface. The sea of electrically charged particles is called a plasma, and it is accelerated away from the pulsar by the electromagnetic fields. This outflow is called the *pulsar wind*. The pulsar wind is extremely relativistic, with particles moving at speeds very close to the speed of light.

The traditional view of the pulsar wind is that, like the solar wind from our Sun, its properties at any point do not change with time. It is said to be a steady-state wind. There are some problems with this model. The most obvious one is that a giant rotating electromagnet should cause the pulsar wind to oscillate at the rotation frequency in some way.

The wind contains both an electromagnetic and a kinetic energy component. Near the pulsar, the electromagnetic component is believed to dominate the energy flow. Steady-state wind theory predicts that the electromagnetic component will also dominate at large distances from the pulsar, where the wind slams into the surrounding nebula. However, observations show that the opposite is true — kinetic energy must dominate at this boundary in order for the expansion speed and pressure at the edge of the nebula to match the values observed (Kennel & Coroniti, 1984). This inconsistency is known as the σ paradox.

Also, recent Hubble Space Telescope images have shown that the wind is not steady-state (Hester et al, 1995; www.stsci.edu) — the shock where the wind strikes the nebula has wisp-like structures varying on a time-scale of days.

The alternative model is that the wind is wave-like. The wave is not a mere ripple disturbing the charges; it is more like a giant tidal wave hurling the charges around with immense force. It is strong enough to accelerate the charges to ultra-relativistic speeds.

My mentor has proposed (Melatos & Melrose, 1996) a wave-like model which correctly predicts the oscillating nature of the wind and resolves the σ paradox. For this model to be viable, it must be stable.

2 The Stability of Wave-Like Pulsar Winds

The problem with most wave-like models is their stability. Unlike a steady state wind, a wave-like wind loses energy because the charges that make up the wind are accelerated by the electromagnetic fields and radiate power. The wave is re-energised each cycle as the pulsar rotates, so if the wind loses all its energy on a timescale shorter than the pulsar period then the wave-like model is unstable.

Previous work (Asseo et al, 1978) has shown that intense relativistic plasma waves are damped by radiation under certain circumstances. Our aim is to investigate damping in the context of pulsar winds.

The wave can also be destroyed by the amplification of perturbations (Max & Perkins, 1972; Sweeney & Stewart, 1978) due to hydromagnetic instabilities. This was investigated by another 1997 SURF student, Ronak Bhatt.

As part of this SURF project, I solved the equations in Section 2.1, in the limit of negligible damping, to obtain a solution for the wind properties. Using this solution and Larmor'sformula for the power radiated by an accelerated charge, I calculated the effect of the damping.

2.1 Plasma Equations

A cold e^-e^+ plasma obeys the equations

$$\frac{\partial n^\pm}{\partial t} + \mathrm{div}(n^\pm \mathbf{v}^\pm) = 0 \ , \tag{1}$$

$$\frac{\partial \mathbf{p}^\pm}{\partial t} + (\mathbf{v}^\pm.\ \mathrm{grad})\mathbf{p}^\pm = \pm e(\mathbf{E} + \mathbf{v}^\pm \times \mathbf{B}) + \mathbf{RR}^\pm \ , \tag{2}$$

$$\mathbf{RR}^\pm = \frac{e^2}{6\pi\epsilon_o c^3}\left\{\gamma^\pm \frac{d}{dt}\left[\gamma^\pm \frac{d}{dt}(\gamma^\pm \mathbf{v}^\pm)\right] - \frac{\gamma^{\pm 4}}{c^2}\left[\frac{\gamma^{\pm 2}}{c^2}\left(\mathbf{v}^\pm . \frac{d\mathbf{v}^\pm}{dt}\right)^2 - \left(\frac{d\mathbf{v}^\pm}{dt}\right)^2\right]\right\}\gamma^\pm \mathbf{v}^\pm \ , \tag{3}$$

$$\mathrm{div}\mathbf{E} = \frac{e}{\epsilon_o}(n^+ - n^-) \ , \tag{4}$$

$$\mathrm{div}\mathbf{B} = 0 \ , \tag{5}$$

$$\mathrm{curl}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \ , \tag{6}$$

$$\mathrm{curl}\mathbf{B} = \mu_o e(n^+\mathbf{v}^+ - n^-\mathbf{v}^-) + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \ , \tag{7}$$

where the symbols have their usual meaning. $\mathbf{RR}^\pm$ is the Radiation Reaction force; it is the rate at which momentum is carried away by the radiation field of a radiating charge.

2.2 Undamped Wind Solution

I assumed travelling wave solutions depending on $\mathbf{x}$ and t through $(\omega t - \mathbf{k}.\mathbf{x})$. If the Radiation Reaction force is neglected, then these equations can be solved to find the velocities of the plasma particles and all other wind properties. We found that the path of the plasma particles is a helix, with their speeds along the axis greatly exceeding their speeds of rotation around the axis. The presence of a background magnetic field causes the electrons and positrons to rotate at different speeds; those rotating in the same sense as their gyration rotate fastest. Two more constraints are imposed on the wind solution.

- There must not be a buildup of particles at any point in the wind. The pulsar injects particles into the wind at a constant rate, so the number density of the particles (averaged over many wave cycles) must fall off as an inverse-square law with distance from the pulsar.
- There must not be a buildup of energy at any point in the wind. The pulsar injects energy into the wind at a constant rate, so the energy density of the particles (averaged over many wave cycles) must fall off as an inverse-square law with distance from the pulsar.

I found unique solutions to (1) — (7) and these two constraints via a graphical technique implemented on Mathematica.

2.3 Damping by Radiation

The above solution, valid if the plasma does not radiate energy, is said to be undamped. The energy that should be lost to radiation can now be calculated, using this solution and Larmor's formula for the power radiated by an accelerated charge. By comparing this radiated power to the energy present in the wind, we are able to calculate the time it would take for the wind to lose all its energy — the damping time. If the damping time is greater than one period, then the wind — which is re-energised every period by the rotating pulsar — is stable.

Using the following pulsar parameters, we obtained the damping time as a function of distance from the pulsar: number of particles emitted per second by the pulsar = 1.5×10^{38}, energy lost per second by the pulsar = 5×10^{31} Joules, Lorentz factor for the speed at which particles travel away from the pulsar = 4×10^{6}.

The results (Figure 1) show that the wave-like model is stable, indeed it is extremely stable. The inverse-square dependence exhibited by the damping time is also interesting. One might have expected the damping time to increase as the wave moves away from the pulsar, as the intense fields weaken and the acceleration is less. The power radiated is proportional to the square of the acceleration, so this reduces the energy loss.

Why are the intense relativistic plasma waves in a pulsar wind so stable, unlike cases studied previously? The main new feature in this model is *streaming*, the transport of particles away from the pulsar. The (constant) streaming velocity (along the

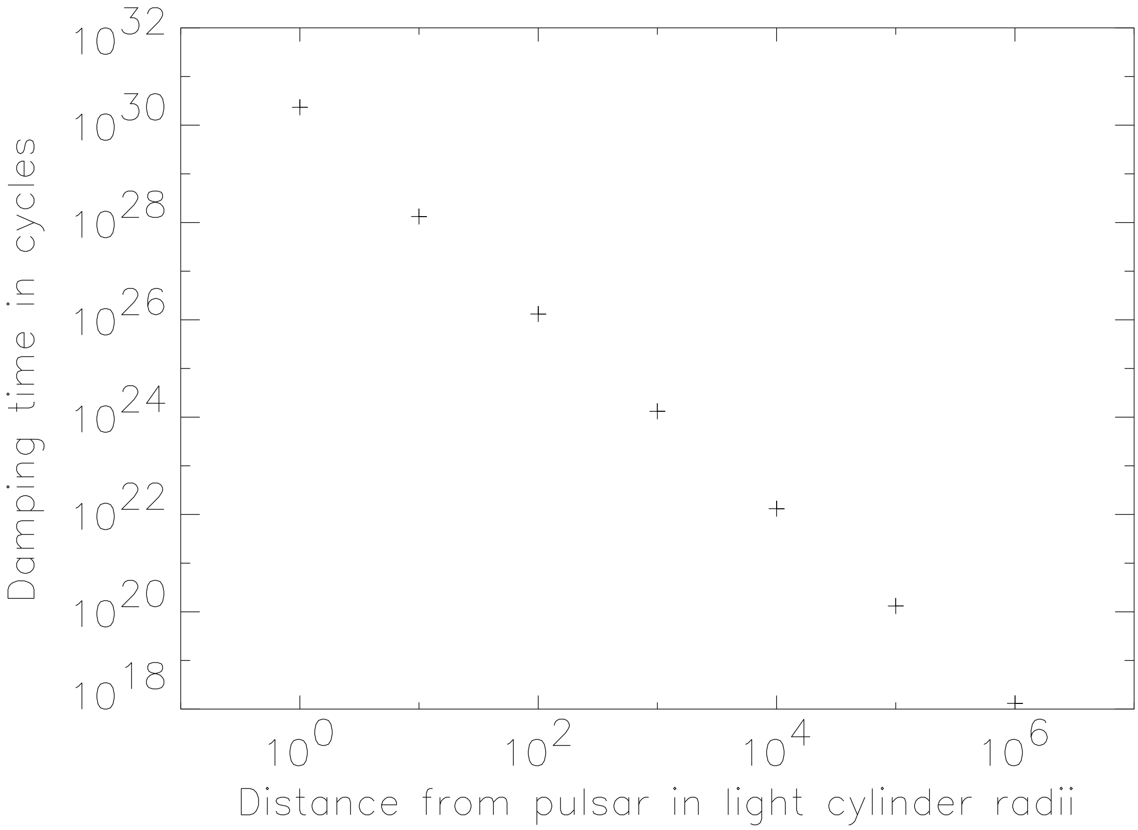

Figure 1: Damping time for the equilibrium solution as a function of distance from the pulsar, measured in terms of a characteristic distance from the pulsar.

direction of propagation) is very large, yet the speed of rotation around the direction of propagation (and consequent acceleration) is very small. This makes the radiated power much smaller than in previous work. The wave-like model has a characteristic dependence on position and time, and we have found that, for fast streaming, this introduces an additional term into the Radiation Reaction Force which almost exactly cancels with the usual term. These two effects together make the Radiation Reaction force negligible and the damping time very large.

3 Conclusion

The σ paradox is a major problem for steady-state models of pulsar winds. For a wave-like model to be viable as an alternative, it must be stable. We have constructed an equilibrium solution for the wind as a wave, which conserves energy and particle number, and is extremely stable to radiation losses. The streaming of the wind away from the pulsar is crucial in stabilising the wave.

My mentor and I are currently writing a Fortran program to integrate (1) — (7) numerically. By choosing initial conditions, corresponding to the equilibrium solution determined analytically (Section 2.2), and then perturbing them slightly, we intend to verify the large radiative damping times shown in Figure 1 and the analytic results of Bhatt under various hydromagnetic and radiative instabilities.

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