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A smoothing technique for improving atmospheric reconstruction for planetary entry probes

Paul Withers *

Department of Astronomy, Center for Space Physics, Boston University, 725 Commonwealth Avenue, Boston, MA 02215, USA

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ABSTRACT

Accelerometer instruments are commonly used on planetary entry probes to determine vertical profiles of atmospheric density, pressure, and temperature. A key consideration for the design and implementation of such investigations is extending the atmospheric results to the highest altitudes possible, which requires minimizing uncertainties in measured accelerations whilst maintaining adequate vertical resolution. Since atmospheric density depends exponentially on altitude, the arithmetic mean of a subset of raw acceleration data points is a biased estimate of the true acceleration at the center of the time interval in question. This diminishes the quality of derived atmospheric properties. Here we show how this problem can be alleviated by using a specialized averaging technique that takes advantage of the inherent exponential variation in acceleration with time at atmospheric entry. This technique is demonstrated successfully on Mars Phoenix data.

1. Introduction

A spacecraft descending into a planetary atmosphere will often operate an accelerometer during atmospheric flight (Desai et al., 2011). Post-flight, these acceleration measurements can be used to determine the spacecraft's atmospheric trajectory and to analyze vehicle performance (Blanchard and Desai, 2011). They can also be used to derive the local atmospheric density along the trajectory, which is scientifically valuable to do (Withers and Catling, 2010). The higher in altitude the reconstructed density profile extends, the more scientifically valuable it is. A long-standing challenge in extending atmospheric density profiles to the highest possible altitude is that the signal, the atmospherically induced acceleration, decreases rapidly with increasing altitude and eventually falls below the instrumental noise limit. Averaging the acceleration measurements improves the signal-to-noise ratio, but at the cost of reduced vertical resolution, which is not scientifically desirable. Here we present a data analysis technique for maintaining an acceptable signal-to-noise ratio and vertical resolution throughout an atmospheric structure reconstruction.

2. Statement of problem

Consider the Phoenix spacecraft, which landed on Mars on 25 May 2008 (Smith et al., 2009). Its accelerometer recorded 200 Hz data during atmospheric entry, descent, and landing. The component of acceleration along the spacecraft's symmetry axis, the axial acceleration, is shown in Fig. 1. As a result of the effects of instrument digitization and noise, these acceleration measurements do not accurately represent the actual acceleration experienced by Phoenix at times before about 1940 s. Atmospheric density is proportional to axial acceleration and can be determined by analysis of the axial acceleration (Magalhães et al., 1999). Here the scatter in the acceleration measurements indicates that atmospheric densities cannot be determined accurately for times prior to 1940 s (altitudes greater than 65 km). Clearly, this problem can be alleviated by averaging the data. However, the choice of averaging interval has consequences. Too short an interval does not extend the reconstructed densities to the greatest altitude possible. Too long an interval reduces vertical resolution, which reduces the scientific value of the results. A variable averaging interval would permit a long interval at high altitudes, where it is needed, to shrink to a shorter, scientifically better interval at lower altitudes where the long interval is unnecessary. However, a variable averaging interval causes complications at the times when it varies. This point is illustrated in Fig. 2, where two different intervals are used to smooth the data shown in Fig. 1. The two sets of smoothed data are different and at least one of them must be unrepresentative of the true accelerations imposed on the spacecraft by the atmosphere. A gradual transition from one smoothed data series to the other could be imposed by taking a weighted average of the two, but that affects the rate of change of the inferred acceleration. This is undesirable, since this rate of change is directly proportional to

* Tel.: +1 617 353 1531; fax: +1 617 353 6463.
E-mail address: withers@bu.edu

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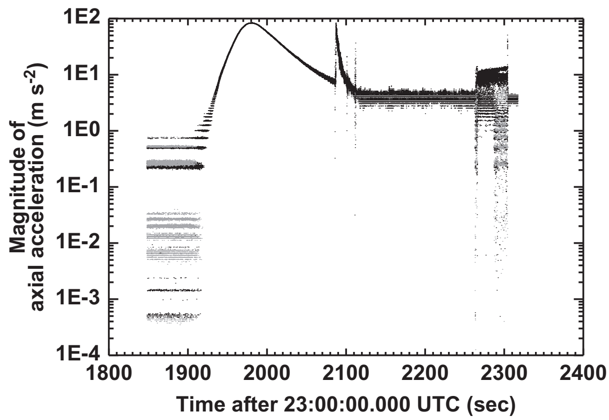

Fig. 1. Magnitude of axial acceleration measurements before smoothing or correcting. Black dots indicate positive accelerations and gray dots indicate negative accelerations. Key features are peak deceleration at 1980 s, parachute deployment at 2090 s, and thruster ignition at 2260 s. Early acceleration measurements, both positive and negative, show effects of instrument digitization and clearly should not be taken at face value. Accelerations depend exponentially on time between 1940 and 1960 s.

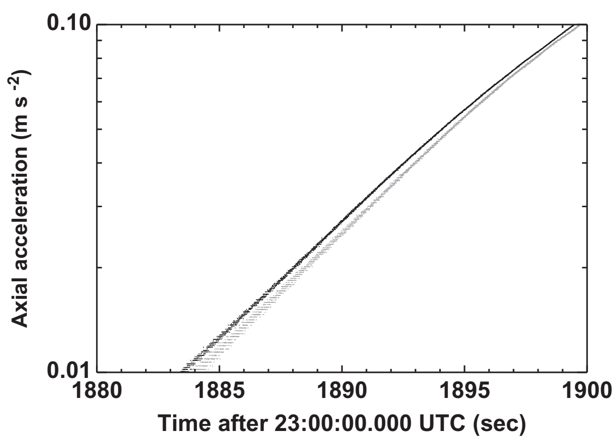

Fig. 2. Two time series of smoothed axial acceleration measurements without correction. Gray dots indicate smoothing with a 1024 point (5.12 s) running mean and black dots indicate smoothing with a 2048 point (10.24 s) running mean. The two data series are clearly distinct, which indicates that at least one of them is inaccurate.

the scientifically valuable atmospheric temperature that can be determined from the reconstructed densities (Withers et al., 2003).

3. Method

Technical details concerning the application of the following method to Phoenix data were reported by Withers et al. (2010) and the corresponding scientific results were reported by Withers and Catling (2010). Readers are directed to those papers for further information.

Atmospheric density, ρ, is related to axial acceleration, a, as follows (Magalhães et al., 1999):

$$ma = \frac{\rho A v^2 C}{2} \tag{1}$$

where m is the spacecraft mass, A is the reference area of the spacecraft, and C is the axial force coefficient, which is usually on the order of two (Withers et al., 2003). At high altitudes where low signal-to-noise is a problem, m, A, v, and C are effectively constant. Under such conditions, a is directly proportional to ρ. Since ρ depends exponentially on altitude, z, as $\rho = \rho_R \exp(-(z-z_R)/H)$, where ρ_R is the density at reference altitude z_R and H is a scale height, and the spacecraft is descending at a constant rate, the acceleration increases exponentially with time. This can be seen in Figs. 1 and 2. The exponential dependence of density on altitude is only approximately satisfied over a finite vertical range in a real atmosphere and we shall later explore the effects of this approximation. Therefore

$$a = a_0 \exp \frac{(t-t_0)}{\tau} \tag{2}$$

where t is time, a_0 is the acceleration at time t_0, and τ is a timescale, which equals the ratio of the atmosphere's density scale height to the rate of change of altitude. The arithmetic mean of the acceleration between $t = t_0 - t_X$ and $t = t_0 + t_X$, a_{mean}, is not the same as the desired acceleration at the center of the time interval unless $t_X \ll \tau$

$$a_{mean} = a_0 \frac{\tau}{t_X} \sinh\left(\frac{t_X}{\tau}\right) \tag{3}$$

Hence each of the two smoothed data series shown in Fig. 2 is offset from the true acceleration by a constant factor, and that factor depends on the size of the averaging interval. Eq. (2) shows that an unbiased estimate of a_0 could be found from the arithmetic mean of the logarithm of a series of acceleration measurements. In this case, no issues arise when a variable averaging interval is used. However, this approach cannot be used when some of the measured accelerations are negative, since their logarithms do not exist. As the abundant gray dots at early times in Fig. 1 indicate negative measurements are common. Therefore an alternative technique is needed.

Suppose that a "long" average, a_L, and a "short" average, a_S, are calculated over the ranges $t = t_0 - 2t_S$ to $t = t_0 + 2t_S$ and $t = t_0 - t_S$ to $t = t_0 + t_S$, respectively. It follows from Eq. (3) that the ratio of these two averages satisfies:

$$\frac{a_L}{a_S} = \cosh\left(\frac{t_S}{\tau}\right) \tag{4}$$

Eq. (4) can be rearranged using the trigonometric identity $\cosh^{-1}(x) = \ln(x + (x^2-1)^{1/2})$ to yield

$$\frac{t_S}{\tau} = \ln\left(\frac{a_L}{a_S} + \sqrt{\left(\frac{a_L}{a_S}\right)^2 - 1}\right) \tag{5}$$

Thus the timescale τ can be determined from the two related means, a_L and a_S. Now the true acceleration at the center of the averaging interval, a_0, can be found using τ, a_S, and Eq. (3). The averaged acceleration can be corrected to a better and unbiased estimate of its true value. Fig. 3 shows that this procedure eliminates the difference between the two smoothed data series that is present in Fig. 2. Note the importance of the factor of two difference between the time intervals. If a different factor is used, then t_L cannot be eliminated from the ratio a_L/a_S (Eq. (4)), which makes the procedure useless.

The selection of an appropriate value for t_S is clearly important. Specification by the user of a desired signal-to-noise ratio and/or vertical resolution is sufficient to determine the appropriate value of t_S at any given time. The assumption of a perfectly exponential atmosphere is only approximately satisfied, and so it is important that the chosen value of t_S be as small as possible relative to τ, subject to the other constraints. In particular, having a very large value of t_S solely to have very, very small formal uncertainties on the corrected acceleration is unwise. Small-scale atmospheric structures that are passed through in a time less than t_S will be smoothed away by this technique; alternative data processing techniques should be used to study such features. If the selected value of t_S results in $a_L < a_S$ at any point, then Eq. (5) involves the

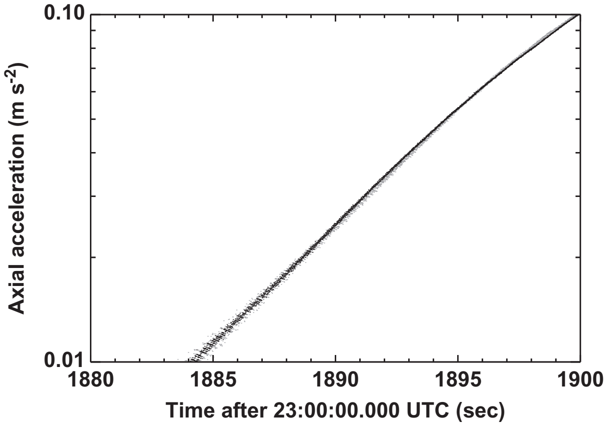

Fig. 3. Two time series of smoothed axial acceleration measurements with correction. Gray dots indicate smoothing with a 1024 point (5.12 s) running mean followed by correction using ratio to a 2048 point (10.24 s) running mean. Black dots indicate smoothing with a 2048 (10.24 s) point running mean followed by correction using ratio to a 4096 (20.48 s) point running mean. The two data series appear indistinguishable.

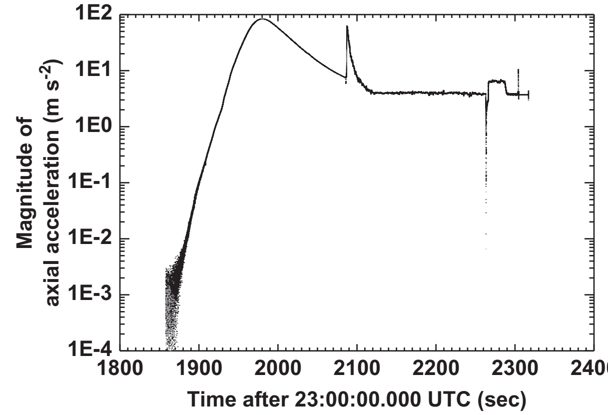

Fig. 4. Magnitude of axial acceleration measurements after smoothing and correcting. Black dots indicate positive accelerations and gray dots indicate negative accelerations. Useful measurements extend to much earlier times than in Fig. 1.

square root of a negative number and the technique fails. If this occurs, increasing t_S should alleviate the problem.

If a specified signal-to-noise ratio is used to determine t_S, then the suggested value of t_S will decrease as time increases and the spacecraft descends deeper into the atmosphere. Using the value of t_S that was appropriate for the upper atmosphere in the lower atmosphere is overkill; by using a smaller t_S, errors caused by the assumption of a perfectly exponential atmosphere could be reduced and the vertical resolution could be improved. Since the uncertainties in the resultant atmospheric properties are affected by uncertainties in the entry state and the aerodynamic database, striving for infinitesimally small formal uncertainties on the acceleration by using very large values of t_S is not worthwhile. A better approach is to use a large value of t_S in the upper atmosphere and a smaller value in the lower atmosphere. In this case, three averaged accelerations can be calculated over the entire data series, a "long" average, a_L, a "short" average, a_S, and a "tiny" average, a_T, where the averaging time interval for a_L is twice that for a_S, which in turn is twice that for a_T. Two time series of corrected accelerations can be found for the entire trajectory, one based on a_L/a_S and one based on a_S/a_T. The former uses longer averaging intervals than the latter. A trajectory reconstruction can be performed adopting the a_L/a_S corrected accelerations at high altitudes and the a_S/a_T corrected accelerations at low altitudes. Fig. 3 indicates that the transition between the two corrected accelerations at the selected time will be smooth.

Of course, this concept can be extended to many different averaging intervals, each related by a factor of two. It is convenient to use averaging intervals given by $\Delta t \times 2^n$, where Δt is the interval between raw data points and n is a positive integer. Using this technique, the 200 Hz acceleration measurements shown in Fig. 1 were smoothed and corrected using an averaging interval that shrank with increasing time and decreasing altitude to produce the results shown in Fig. 4. Figures in Withers et al. (2010) show altitude as a function of time, the temporal/vertical extent of the averaging interval as a function of altitude, and other related plots. The inferred values of τ were, as predicted, consistent with the ratio of the scale height to the descent speed. The averaging interval shrank by successive factors of two from an initial value of 20.48 s ($2^{12} \times 0.005$ s) to 0.32 s ($2^6 \times 0.005$ s) at 1900 s (about 100 km altitude) and remained at 0.32 s thereafter. Now atmospheric densities can be found up to 128 km altitude, a tremendous improvement over the original 65 km threshold. The initial averaging interval used at the top of the atmosphere corresponded to a vertical resolution of 20 km, so the high altitude results provide only a very large-scale view of the atmosphere. Note that, even though 20 km is about twice the atmospheric scale height, the results are still physically realistic (Withers and Catling, 2010; Withers et al., 2010). The value of 20.48 s was chosen to maximize the height to which reasonable averaged acceleration measurements extended. The averaging interval was not decreased below 0.32 s since this corresponded to a vertical resolution of 0.5 km or less and smaller-scale features were not of interest.

A fortuitous feature of this technique is that it fails gracefully. It assumes that acceleration depends exponentially on time, which ceases to be valid after the initial stages of entry. How then have we been able to apply this technique at all times shown in Fig. 4? The solution lies in Eq. (3). When $t_X \ll \tau$, then $a_{mean} = a_0(1+O(t_X/\tau)^2)$. In this case, the value of τ inferred from comparison of different averages is irrelevant. It does not affect the value of a_0 inferred from a_{mean} because, for all practical purposes, $a_{mean} = a_0$. In Fig. 4, an averaging interval of 0.32 s was used after 1900 s, when the timescale for traversing one atmospheric scale height is at least 20 times greater.

4. Conclusions

A technique has been developed to increase the vertical extent of atmospheric density profiles from atmospheric entry probes. It uses long averaging intervals at high altitudes, where measured accelerations are small, shorter averaging intervals at lower altitudes, where longer intervals are unnecessary and scientifically undesirable, and does not introduce errors at the places where the averaging interval changes. It requires that acceleration depend exponentially on time, which is generally satisfied at high altitudes. When this requirement is not satisfied, a typical accelerometer is sufficiently sensitive that this specialized technique is not necessary. This technique has been successfully demonstrated on Phoenix data and could be used to reanalyze data from other atmospheric entry probes.

Acknowledgments

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