

A technique to determine the mean molecular mass of a planetary atmosphere using pressure and temperature measurements made by an entry probe: Demonstration using Huygens data

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Abstract

It is possible to determine the mean molecular mass of a planetary atmosphere using pressure and temperature measurements made by an entry probe descending at terminal velocity. The descent trajectory of an entry probe can be determined from pressure, temperature, and mean molecular mass data. This technique offers redundancy for large entry probes in the event of a mass spectrometer failure and increases the potential scientific yield of small entry probes that do not carry mass spectrometers. This technique is demonstrated on Huygens atmospheric structure instrument (HASI) data from Titan. Accurate knowledge of entry probe and parachute drag coefficients is required for this technique to be useful.

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1. Introduction

Many entry probe missions have carried a mass spectrometer. One of the purposes of such experiments is to determine the mean molecular mass of the planetary atmosphere during the probe's descent beneath a parachute. Once this is determined, the probe's descent trajectory can be reconstructed using time series of mean molecular mass, pressure, and temperature (Young et al., 1996; Seiff et al., 1998; Lebreton et al., 2005; Fulchignoni et al., 2005). Mass spectrometers are amongst the heaviest, most delicate, and most expensive instruments on an entry probe, whereas pressure and temperature sensors are amongst the lightest, most robust, and cheapest. The sampling rate of a pressure or temperature sensor is typically much greater than that of a mass spectrometer

(Niemann et al., 1992, 2002; Seiff and Knight, 1992; Fulchignoni et al., 2002).

The aim of this paper is to develop and demonstrate a technique by which the mean molecular mass of a planetary atmosphere can be determined using pressure and temperature measurements made by an entry probe descending at terminal velocity. Related techniques have been used to determine the mean molecular mass of the terrestrial atmosphere (Sommer and Yee, 1969; Seiff et al., 1973).

2. Method

The ideal gas equation of state is

$$p = \frac{\rho R_u T}{\mu} \quad (1)$$

where p is pressure, ρ is density, R_u is the universal gas constant, T is temperature, and μ is the mean molecular mass. The equation of hydrostatic equilibrium is

$$\frac{dp}{dz} = -\rho g, \quad (2)$$

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where z is altitude and g is the magnitude of the acceleration due to gravity, which satisfies

$$g = g_0 \left(\frac{R}{R+z} \right)^2, \quad (3)$$

where g_0 is the value of g at $z = 0$ and R is the planetary radius. Given an initial value for z and probe measurements of $p(t)$, $T(t)$, and $\mu(t)$, where t is time, these equations can be used to obtain $z(t)$ and $\rho(t)$. This approach has been used on many previous entry probe missions (Seiff et al., 1980, 1998; Fulchignoni et al., 2005). If $\mu(t)$ is not known, then neither $z(t)$ nor $\rho(t)$ can be found without an additional constraint. After an initial hypersonic entry within a protective aeroshell, a typical entry probe descends at terminal velocity beneath a parachute and its speed satisfies

$$mg = \frac{\rho A v^2 C_D}{2}, \quad (4)$$

where m is the mass of the probe and parachute, A is the cross-sectional area of the parachute, v is defined shortly, and C_D is a dimensionless drag coefficient. A and m are known constants and g is a known function of altitude. I shall assume that C_D is constant. In theory, C_D depends weakly on the Mach and Reynolds numbers during the descent (e.g. Jaremenko et al., 1971; Huygens project document MBA-SYS-RE-200301). The technique outlined here can be extended to cases in which C_D is not constant. In Eq. (4), v is formally the speed of the probe relative to the atmosphere. A relationship between v and z is required for further progress. If the probe's horizontal motion and vertical winds can be neglected, then v and z satisfy

$$v = \frac{dz}{dt}. \quad (5)$$

Thus v is negative for a descending probe. Given measurements of $p(t)$ and $T(t)$ and initial values of v and z , $z(t)$, $v(t)$, $\mu(t)$, and $\rho(t)$ can be determined from Eqs. (1)–(5). Let $Q = dp/dt$. If Eqs. (2), (4), and (5) are rearranged to eliminate ρ and discretized, then

$$v_{i+1} = \frac{-2mg_0^2}{Q_i A C_D} \left(\frac{R}{R+z_i} \right)^4, \quad (6)$$

$$z_{i+1} = z_i + (t_{i+1} - t_i)v_i, \quad (7)$$

where i is an index. Once $z(t)$ and $v(t)$ have been found using Eqs. (6) and (7), Eq. (2) can be used to find $\rho(t)$ and Eq. (1) can be used to find $\mu(t)$. Another method for determining μ without a mass spectrometer is offered by instruments that measure the speed of sound, c , which satisfies (e.g. Zarnecki et al., 2002; Kazeminejad and Atkinson, 2004)

$$c = \sqrt{\left(\frac{\gamma R_u T}{\mu} \right)}, \quad (8)$$

where γ , the ratio of the specific heat capacity at constant pressure to the specific heat capacity at constant volume, is

generally between 1 and 2 (Atkins, 2002). If both c and T are measured, the ratio μ/γ can be found. If sufficient knowledge about the likely atmospheric constituents is available, then it might be possible to determine μ from μ/γ unambiguously.

3. Application to Huygens data

The Huygens atmospheric structure instrument (HASI) recorded pressure and temperature during the parachute descent of Huygens to the surface of Titan (Fulchignoni et al., 2005). The HASI team used Huygens GCMS measurements of $\mu(t)$ and Eqs. (1)–(3) to determine altitude and density. Measured pressure (p_{HASI}), measured temperature (T_{HASI}), derived density (ρ_{HASI}), and derived altitude (z_{HASI}) are tabulated as functions of time at ESA's Planetary Science archive (PSA) and NASA's Planetary Data System (PDS) (ftp://psa.esac.esa.int/pub/mirror/CASSINI-HUYGENS/HASI/HP-SSA-HASI-2-3-4-MISSION-V1.1/DATA/PROFILES/HASI_L4_ATMO_PROFILE_DESCEN.TAB). The pressures and temperatures have been corrected for dynamical effects. Derived descent velocities (v_{HASI}) can be determined from tabulated altitudes (Figs. 1 and 2). The HASI team and the Huygens Descent Trajectory Working Group performed independent reconstructions of the probe trajectory, as discussed in the archived PSA/PDS HASI file AAREADME.TXT (Atkinson et al., 2005). I used the technique described above to determine altitude (z), descent velocity (v), density (ρ), and mean molecular mass (μ) as functions of time from the tabulated pressures (p_{HASI}) and temperatures (T_{HASI}). Huygens used three different parachutes (in order of use: drogue, main, and stabiliser) and some time elapsed between deployment of each parachute and the probe decelerating to the appropriate terminal velocity. To simplify the analysis, the time interval studied here extended from the point at which Huygens reached terminal velocity on the stabiliser parachute until impact. The start time was 1040.875 s after Huygens reference time

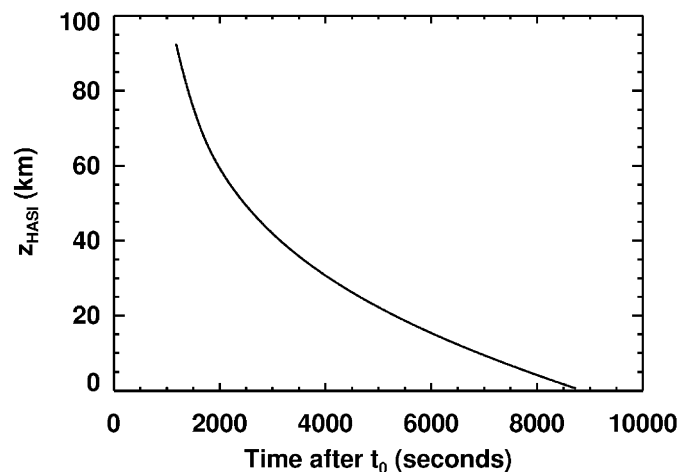


Fig. 1. z_{HASI} (km) versus time.

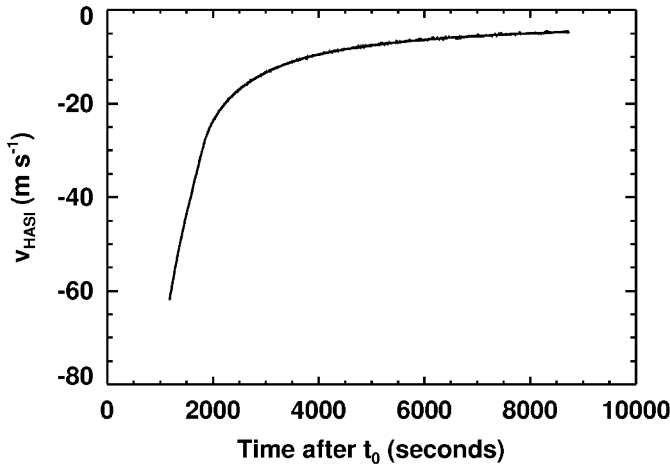


Fig. 2. v_{HASI} (m s^{-1}) versus time.

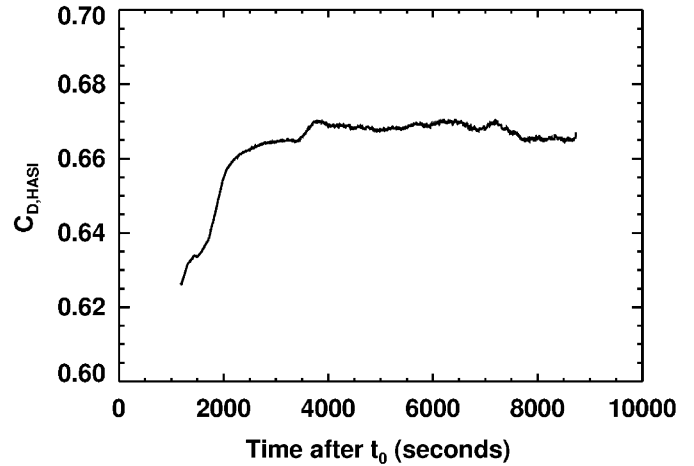


Fig. 3. C_D calculated from using z_{HASI} , v_{HASI} , and ρ_{HASI} in Eqs. (3) and (4).

t_0 , which occurred at 09:10:21 UTC (Lebreton et al., 2005). Initial values of z and v were 101.667 km and -72.1 m s^{-1} . Known parameters are $m = 200.48 \text{ kg}$ (Huygens project document HASI-RP-UPD-105), $A = \pi D^2/4$, $D = 3.03 \text{ m}$ (Jones and Giovagnoli, 1997), $R_u = 8.3144 \text{ J K}^{-1} \text{ mol}^{-1}$, $g_0 = 1.35 \text{ m s}^{-2}$, and $R = 2575 \text{ km}$ (Lodders and Fegley, 1998).

Selection of the appropriate constant value for C_D must be considered carefully. Many probe sensors determined the time of impact. The altitude at impact, $z = 0$, is also known. The derived altitude at impact depends on the chosen value of C_D as follows: $C_D = 0.60, 0.64, 0.65, 0.66, 0.70$ leads to $z_{\text{impact}} = -9.5, -2.0, -0.2, 1.4, 7.6 \text{ km}$. It is clear that $C_D = 0.65$ is the best choice. Is this value reasonable? If z_{HASI} , v_{HASI} , and ρ_{HASI} (derived using GCMS measurements of μ) are used in Eqs. (3) and (4), then C_D is 0.63 at the start of the descent and 0.67 at the end of the descent (Fig. 3). An analysis of HASI data by Mäkinen et al. (2006) found that $A C_D/2m = 0.012 \text{ m}^2 \text{ Kg}^{-1}$ or $C_D = 0.67$, and that C_D did not vary significantly during descent. Preflight models predicted $C_D \sim 0.5$, suggesting that such modelling is relatively poor (Huygens project document MBA-SYS-RE-200301).

Differentiation is an intrinsically noisy operation and the derivatives dp_{HASI}/dt and dp_{HASI}/dz contained significant scatter. A 57-point running mean was used to reduce this scatter, requiring that the first and last 28 data points be discarded. The archived HASI results are listed at 5 s intervals, so this smoothing corresponds to a timescale of 285 s and a lengthscale of 2.85 km for a typical velocity of -10 m s^{-1} .

Fig. 4 shows that the absolute value of the difference between $v(t)$ and $v_{\text{HASI}}(t)$ is between 0.5 and 2.0 m s^{-1} for the first 600 s, then less than 0.5 m s^{-1} for the remaining 7000 s. $v(t)$ is greater than (since v is negative, this means a smaller absolute value) $v_{\text{HASI}}(t)$ when the assumed C_D is an overestimate and less than $v_{\text{HASI}}(t)$ when the assumed C_D is an underestimate, consistent with Eq. (6). Fig. 5 shows that the greatest difference between $z(t)$ and $z_{\text{HASI}}(t)$ is about

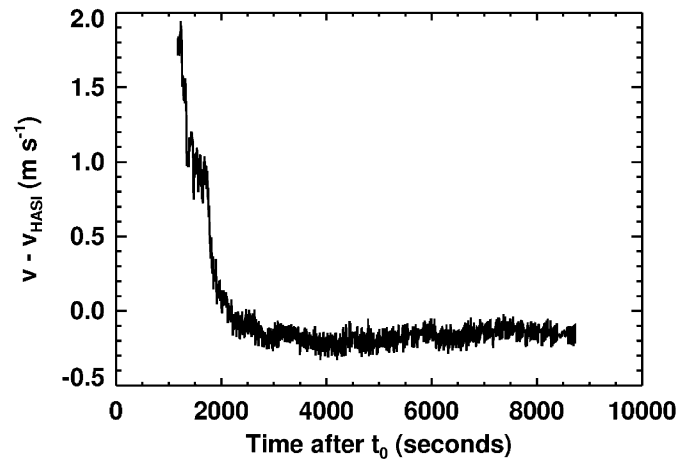


Fig. 4. $v - v_{\text{HASI}}$ (m s^{-1}) versus time.

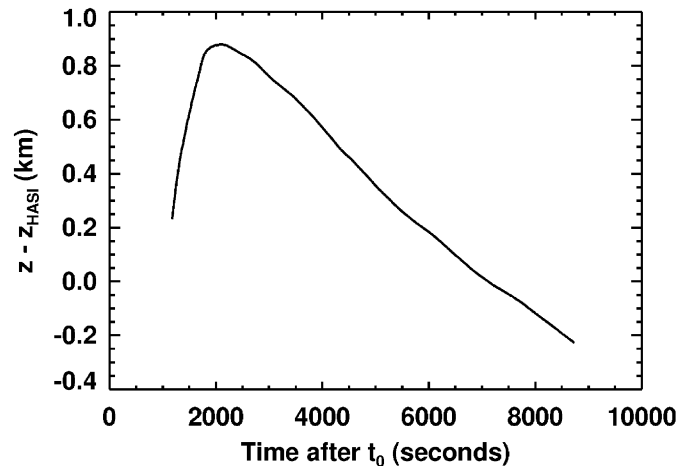


Fig. 5. $z - z_{\text{HASI}}$ (km) versus time.

0.9 km. Trends in Fig. 5 are also related to whether the assumed C_D is an underestimate or overestimate. Fig. 6 shows that $\rho(t)$ is typically less than $\rho_{\text{HASI}}(t)$ by 2–4%. Fig. 7 shows that $\mu(t)$ is between 26.5 and 28.5 Da with a

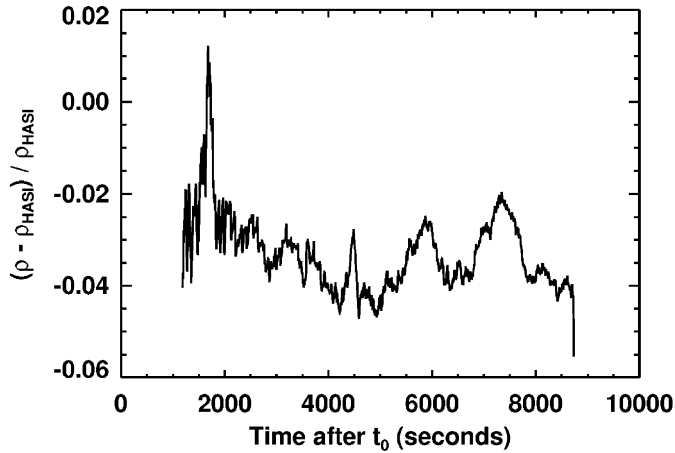


Fig. 6. $(\rho - \rho_{\text{HASI}})/\rho_{\text{HASI}}$ versus time.

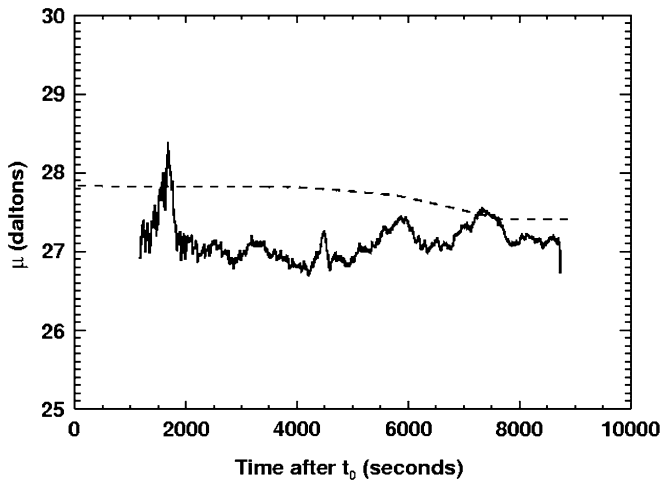


Fig. 7. μ (daltons) versus time (solid line). GCMS results shown as dashed line.

mean value of 27.1 Da, whereas the Huygens GCMS instrument measured $\mu(t)$ between 27.4 and 27.9 Da with a mean value of 27.7 Da (Niemann et al., 2005). The difference between the mean values is 2%. GCMS data were acquired from the PSA (ftp://psa.esac.esa.int/pub/mirror/CASSINI-HUYGENS/GCMS/HP-SSA-GCMS-3-FCO-DESCENT-V1.0/DATA/DTWG_MOLE_FRACTION/GCMS_MOLE_FRACTION_STG2.TAB).

The effects of non-ideal gases can be considered. The equation of state for non-ideal methane–nitrogen mixtures under Titan-like conditions can be reduced to a quadratic equation for μ with one negative (unphysical) and one positive root (Gaborit, 2004). Replacing the ideal gas law with this equation of state increases the derived values of μ by 0.1 Da or less.

Eqs. (1), (2), and (4) can be rearranged to obtain

$$\frac{\mu}{C_D} = \frac{AR_u T(dp/dt)^2}{2mg^3p}. \quad (9)$$

If all other quantities are known perfectly, then the fractional error in μ is the same as in C_D . Whether or not

this technique can be usefully applied to other entry probes depends on how accurately C_D can be determined. Application to giant planet entry probes is dependent on preflight modelling of C_D or additional constraints on the probe's altitude. Venus, Mars, and Titan entry probes that, like Huygens, impact on a surface of known altitude can use this constraint to estimate C_D . The predicted value of C_D for the Huygens stabiliser parachute, ~ 0.5 , is significantly less than the actual value of 0.65. However, predictions of C_D for the main parachute, which was designed to share the same aerodynamic characteristics as the stabiliser parachute, were much more accurate. If the use of this technique is an integral part of a mission, then extensive wind tunnel testing in flight-like conditions should be used to supplement numerical modelling and parametric estimates.

Hydrogen/helium mixing ratios in giant planet atmospheres, and thus mean molecular masses, are uncertain by a few percent (Encrenaz, 2005). The CO_2/N_2 ratio in the Venus atmosphere is uncertain by about one percent, although there is “little observational support” concerning variations in this ratio with altitude (von Zahn et al., 1983). The Huygens GCMS measurements of the CH_4/N_2 ratio in Titan's atmosphere may be more accurate than one percent, but large spatial and temporal variations in this ratio seem possible (Niemann et al., 2005). Preflight modelling of C_D should aim for an accuracy of 1% or better so that results from this technique can distinguish between competing scientific hypotheses concerning the composition and structure of the relevant planetary atmosphere. This is a challenging, but not impossible, goal.

4. Conclusions

The equation of hydrostatic equilibrium, an equation of state, and an equation relating to terminal velocity can be used to determine the mean molecular mass of a planetary atmosphere using pressure and temperature measurements made by an entry probe descending at terminal velocity. Probe altitude, descent velocity, and atmospheric density can also be determined using these three equations. This technique has been successfully demonstrated on HASI data from Titan's atmosphere, showing that the mean molecular mass of Titan's atmosphere is 26.5–28.5 Da, consistent with other measurements. Accurate knowledge of entry probe and parachute drag coefficients is required for this technique to be useful. This technique offers redundancy for large entry probes in the event of a mass spectrometer failure and increases the potential scientific yield of small entry probes that do not carry mass spectrometers. It cannot reproduce the sensitivity of a mass spectrometer nor identify atmospheric species.

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