Attenuation of radio signals by the ionosphere of Mars: Theoretical development and exploratory survey

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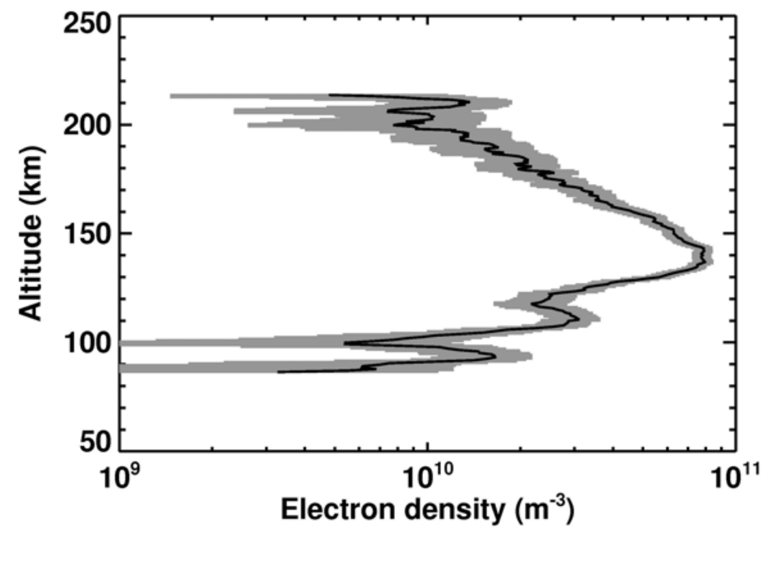
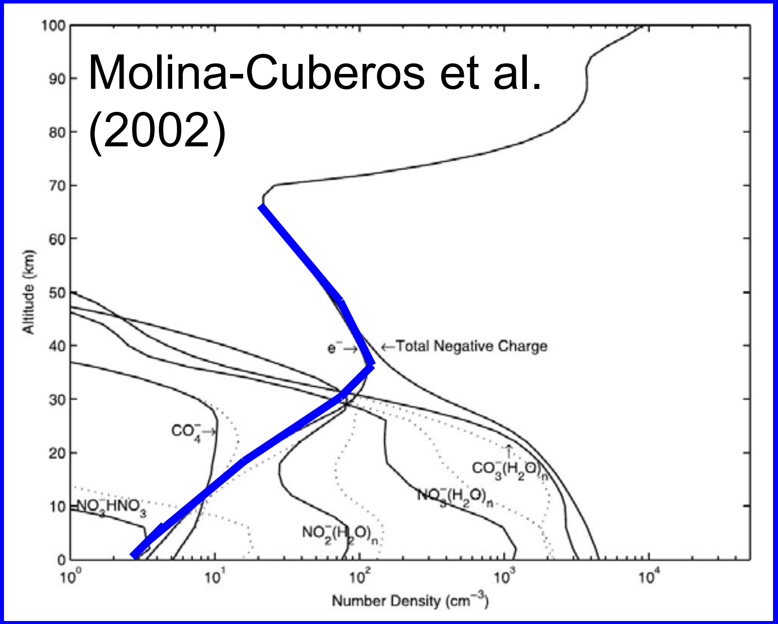
Back to basics

- Develop feel for possible scenarios with theoretical studies of range of idealized situations
- Identify key parameters, functional dependences, scaling laws, order of magnitude estimates
- Tackle specific events once this background is satisfactorily established

Approach

- Idealized neutral atmosphere
  - Uniform scale height and composition
- Background ionosphere as Chapman layer
  - Subsolar N_{max} = 2E11 m^{-3}, z_{max} = 120 km
- Add various additional layers (assume Chapman shape)
  - Secondary photochemical layer at 100 km
  - Meteoric plasma layer at 85 km
  - <u>Predicted</u> plasma layer caused by cosmic rays at 35 km
- Vary radio frequency and peak electron density





Power loss equation

$$\mu^2 = 1 - \frac{Nq^2}{4\pi^2 m_e \epsilon_0 f^2}$$

$$K = \frac{q^2}{2 m_e c \epsilon_0} \cdot \frac{N}{\mu} \cdot \frac{\nu}{\nu^2 + \omega^2}$$

$$P\,(\mathrm{dB}) = -20 \log_{10}(E_r / E_t) = 20 \log_{10}(e) \sec(\mathrm{OZA}) \left(\int K \, dz \right)$$

$$f_p^2 = Nq^2 / 4\pi^2 m_e \epsilon_0$$

and Fp = maximum value of fp

- Effects of multiple plasma layers are additive
- v is electron-neutral collision frequency, proportional to neutral number density
- At high frequencies or altitudes, $w^2 >> v^2$
- At low frequencies or altitudes, $w^2 << v^2$
- Boundary occurs where w = v at z = z_L
  - 64, 47, 29, 12 km for f = 1, 10, 100, 1000 MHz

High frequency limit

- Demonstrate using main ionospheric layer, whose width is the neutral scale height

$$K_{\mathrm{bgd}} = \frac{q^2}{2m_e c \epsilon_0} \cdot \frac{N_{\mathrm{bgd}} \nu}{\omega^2} \cdot \frac{1}{\mu}$$

$$P_{\mathrm{bgd}}\,(\mathrm{dB}) = P_{\mathrm{bgd},0}\,(\mathrm{dB}) \left(1 + \beta_{\mathrm{bgd}} + b_{\mathrm{bgd},2}\beta_{\mathrm{bgd}}^2 + b_{\mathrm{bgd},3}\beta_{\mathrm{bgd}}^4 + O\left(\beta_{\mathrm{bgd}}^4\right)\right)$$

$$\beta_{\mathrm{bgd}} = \frac{\exp\left(0.5\right)}{2\sqrt{2\pi}} \left(\frac{f}{F_{p,\mathrm{bgd}}}\right)^{-2} = 0.329 \left(\frac{f}{F_{p,\mathrm{bgd}}}\right)^{-2}$$

- Lowest order term, $P_{\mathrm{bgd},0}$, is sufficient

$P_{\mathrm{bgd},0}$

$$P_{\mathrm{bgd},0}\,(\mathrm{dB}) = \frac{20\log_{10}(e)\sec(\mathrm{OZA})\,\phi}{2c\sigma Ch}\cdot\exp(0.5)\sqrt{2\pi}\cdot\left(\frac{f}{F_{p,\mathrm{bgd}}}\right)^{-2}$$

$$P_{\mathrm{bgd},0}\,(\mathrm{dB}) = 3.2\times10^{-5}\left(\frac{N_{\mathrm{bgd},0}}{2\times10^{11}\ \mathrm{m}^{-3}}\right)\left(\frac{\phi}{10^{-13}\ \mathrm{m}^3\mathrm{s}^{-1}}\right)\times \left(\frac{\sigma}{3\times10^{-21}\ \mathrm{m}^2}\right)^{-1}\left(\frac{f}{1\ \mathrm{GHz}}\right)^{-2}\frac{\sec(\mathrm{OZA})}{(Ch)^{3/2}}$$

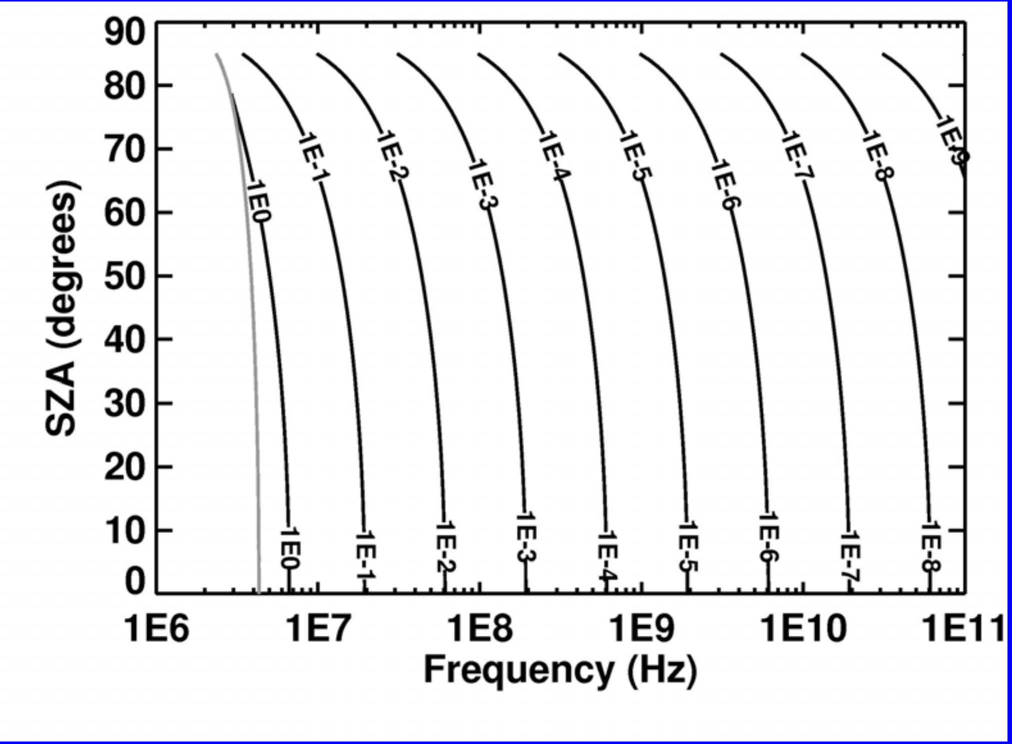
$$P_{\mathrm{bgd},0}\,(\mathrm{dB}) = \frac{2\sec(\mathrm{OZA})}{(Ch)^{1/2}}\left(\frac{f}{F_{p,\mathrm{bgd}}}\right)^{-2}$$

ϕ n = electron-neutral collision frequency defines ϕ

c = speed of light

σ = ionization cross-section

Ch = 1/cos(SZA)



Variations in width of layer at high frequencies

$$N_{XC} = N_{XC,0} \exp\left(\frac{1}{2}\left(1 - \frac{z - z_{XC}}{L_{XC}} - \exp\left(-\frac{z - z_{XC}}{L_{XC}}\right)\right)\right)$$

XC -> “excess”
λ = L_{XC} / H, key ratio
z_{XC} is peak altitude

$$P_{XC,hi}\,(\mathrm{dB}) = P_{XC,hi,0}\,(\mathrm{dB})\left(1 + \beta_{XC,hi} + b_{XC,hi,2}\beta^2_{XC,hi} + b_{XC,hi,3}\beta^4_{XC,hi} + O\left(\beta^4_{XC,hi}\right)\right) \quad (35)$$

$$\beta_{XC,hi} = \frac{\exp(0.5)}{2^{(\lambda_{XC}+3/2)}} \cdot \frac{\Gamma(\lambda_{XC}+1)}{\Gamma(\lambda_{XC}+1/2)} \cdot \left(\frac{f}{F_{p,XC}}\right)^{-2}$$

$$P_{XC,hi,0}\,(\mathrm{dB}) = \frac{20\log_{10}(e)\sec(\mathrm{OZA})\,\phi n_{zXC} H}{2c}\left(\frac{f}{F_{p,XC}}\right)^{-2} \times \lambda_{XC} \cdot \exp(0.5)\, 2^{(\lambda_{XC}+1/2)}\Gamma(\lambda_{XC}+1/2)$$

I will show figures after finding the low frequency limit

- These high frequency theoretical tools are sufficient for layers above 60 km

Low frequency limit

$$K_{XC,lo} = \frac{q^2}{2m_e c\epsilon_0} \cdot \frac{N_{XC}}{\nu} \cdot \frac{1}{\mu}$$

w is irrelevant in this equation

$$K_{XC,lo} = \frac{q^2}{2m_e c\epsilon_0} \cdot \frac{N_{XC,0}}{\phi n_{zXC}} \cdot \frac{1}{\mu}$$
$$\exp\left(\frac{1}{2}\left(1 + \left(2\lambda_{XC} - 1\right) x_{XC} - \exp\left(-x_{XC}\right)\right)\right)$$

$x_{XC} = (z - z_{XC})/L_{XC}$

- If $z_{XC} < z_L$, then $w^2 << v^2$ in all regions with appreciable plasma densities
- Expression for K is infinite at high altitudes, which makes integration difficult
- But there's no plasma at these altitudes, so integrate from low altitudes to $z = z_L$ only
- Since z_L is w-dependent, w returns to the situation

Integrating K in the low frequency limit

$$\int_{z=-\infty}^{z=\infty} K\,dz = \left(\frac{q^2}{2m_e c\epsilon_0}\right)\left(\frac{\lambda_{XC} H N_{XC,0}}{\phi n_{zXC}}\right) \times$$
$$\left[\int_{x=-\infty}^{x=0} \exp\left(\frac{1}{2}\left(1+\left(2\lambda_{XC}-1\right)x-\exp\left(-x\right)\right)\right)\,dx + \int_{x=0}^{x=x_L} \exp\left(\frac{1}{2}\left(1+\left(2\lambda_{XC}-1\right)x\right)\right)\,dx\right]$$

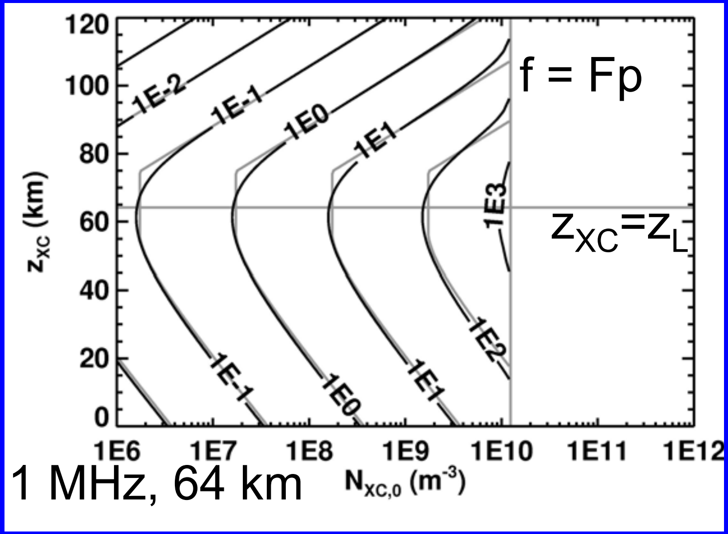
No analytical solution found, so split integral in two and simplify by neglecting part of one term

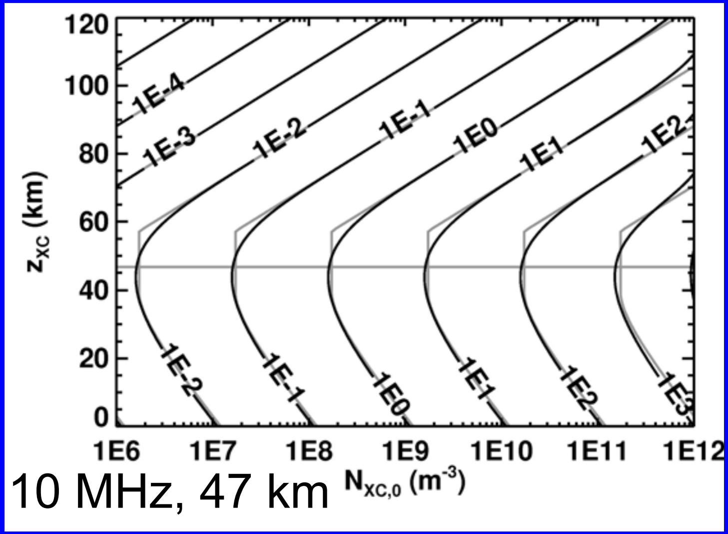
$$P_{XC,lo}\,(\mathrm{dB}) = 20\log_{10}\left(e\right)\sec\left(\mathrm{OZA}\right)\left(\frac{q^2}{2m_e c\epsilon_0}\right)\left(\frac{\lambda_{XC} H N_{XC,0}}{\phi n_{zXC}}\right) \times$$
$$\left[\gamma\left(\lambda_{XC}\right) + \frac{2}{2\lambda_{XC}-1}\exp\left(0.5\right)\left(\exp\left(\frac{\left(2\lambda_{XC}-1\right)x_L}{2}\right)-1\right)\right]$$

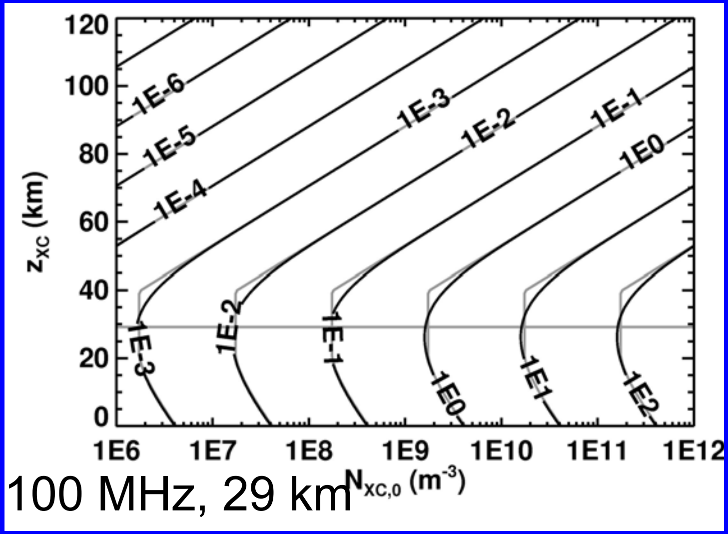
First part of integral represented by γ, which is ~1 for λ~1. Second part has an analytical solution

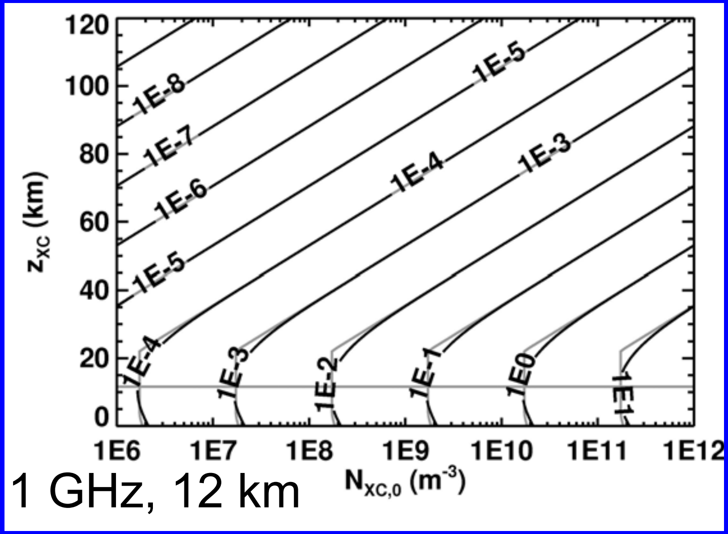
- This is all getting messy – and I haven't even worried about whether refractive index, μ, is exactly 1 or not

Power loss at 1, 10, 100, 1000 MHz

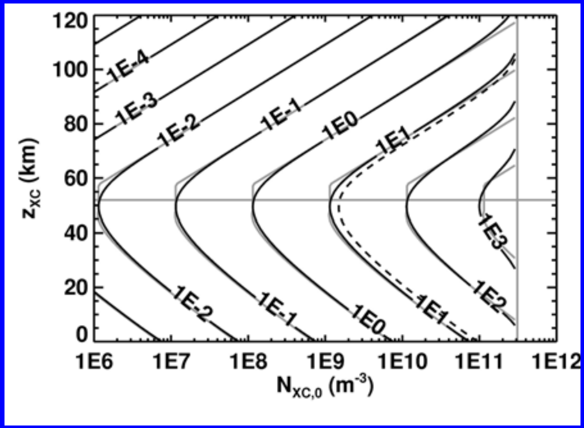




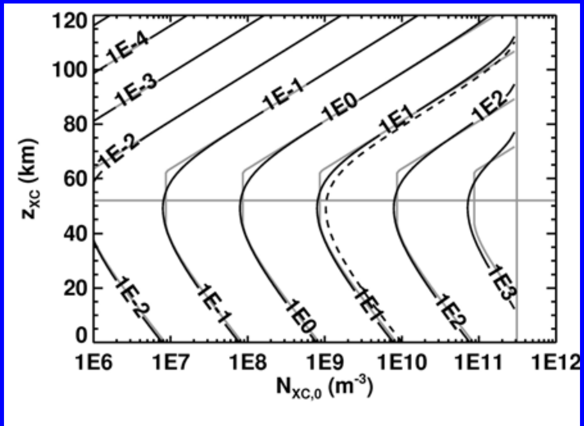




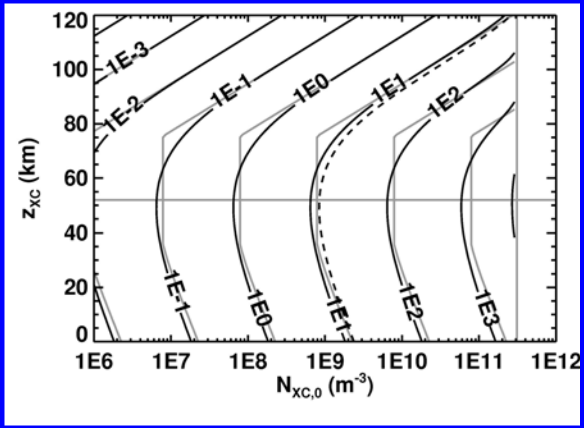
Effect of layer width at f = 5 MHz



λ = 0.5



λ = 1



λ = 2

- Focus on shapes of contours, not values
- Maximum P for given N_{XC} if $z_{XC} = z_L$ (w=v, see next slide)
- Black contours are actual values, grey contours use equations derived on preceding slides. Agreement is pretty good.
- Different slopes for high and low z_{XC}
- λ affects slope of contours for low z_{XC}, but not high z_{XC}

Maximum power loss

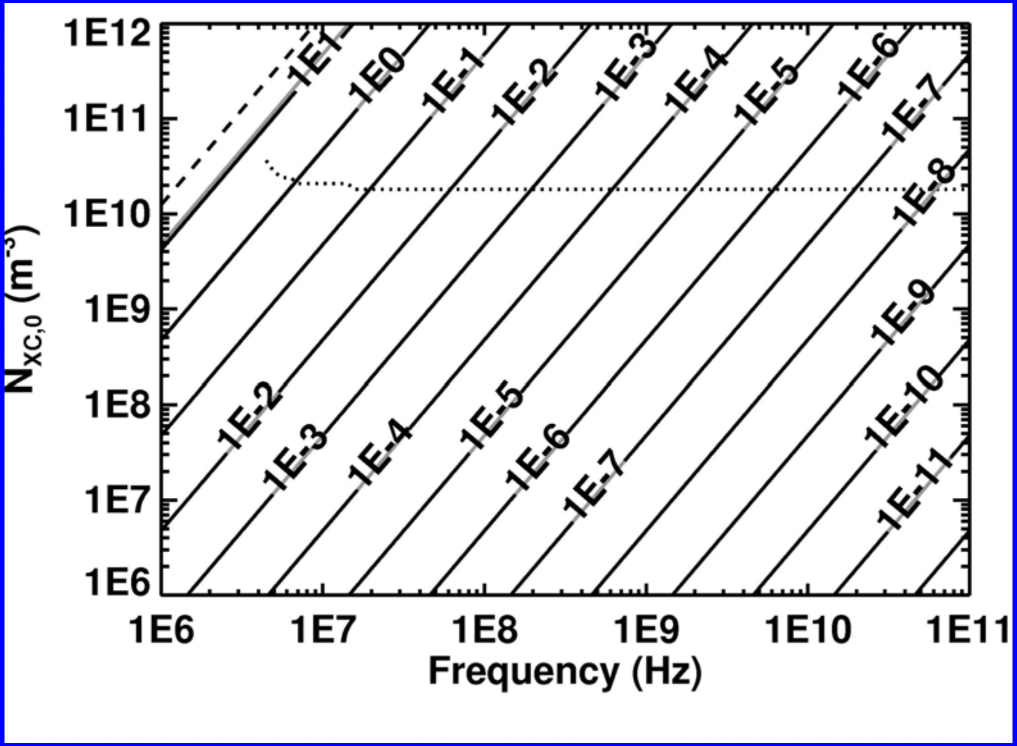
$$P^*_{XC,lo}(\mathrm{dB}) = 20\log_{10}(e)\sec(\mathrm{OZA})\left(\frac{q^2}{2m_e c\epsilon_0}\right)\left(\frac{HN_{XC,0}}{\omega}\right)$$

$$P^*_{XC,lo}(\mathrm{dB}) = 0.73\sec(\mathrm{OZA})\left(\frac{H}{10\ \mathrm{km}}\right)\left(\frac{N_{XC,0}}{10^9\ \mathrm{m}^{-3}}\right)\left(\frac{f}{100\ \mathrm{MHz}}\right)^{-1}$$

P in low frequency limit at $z_{xc} = z_L - \lambda H$

- $z_{XC} > z_L$ $P \propto H\, N_{XC}\, \phi\, n_{zXC}\, f^{-2}$
- $z_{XC} = z_L$ $P \propto H\, N_{XC}\, f^{-1}$
- $z_{XC} < z_L$ $P \propto H\, N_{XC}\, \phi^{-1}\, n_{zXC}^{-0.5}\, f^{-0.5}$

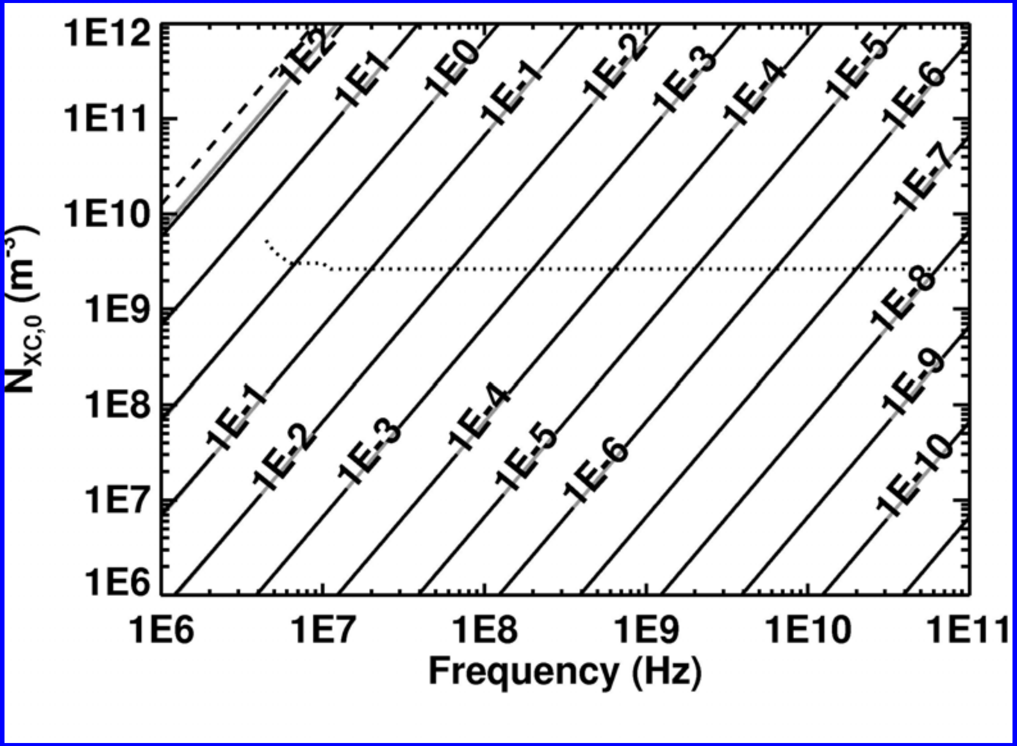
100 km photochemical layer (λ=1)



Dotted black line shows power loss from the main ionospheric layer

- Plausible N_{XC} is 1E11 m^{-3}
- Thus P > 1 dB only for f < 14 MHz
- P exceeds power loss from main ionospheric layer if N_{XC} > 2E10 m^{-3}
- Larger N_{XC} can occur during rare solar flares

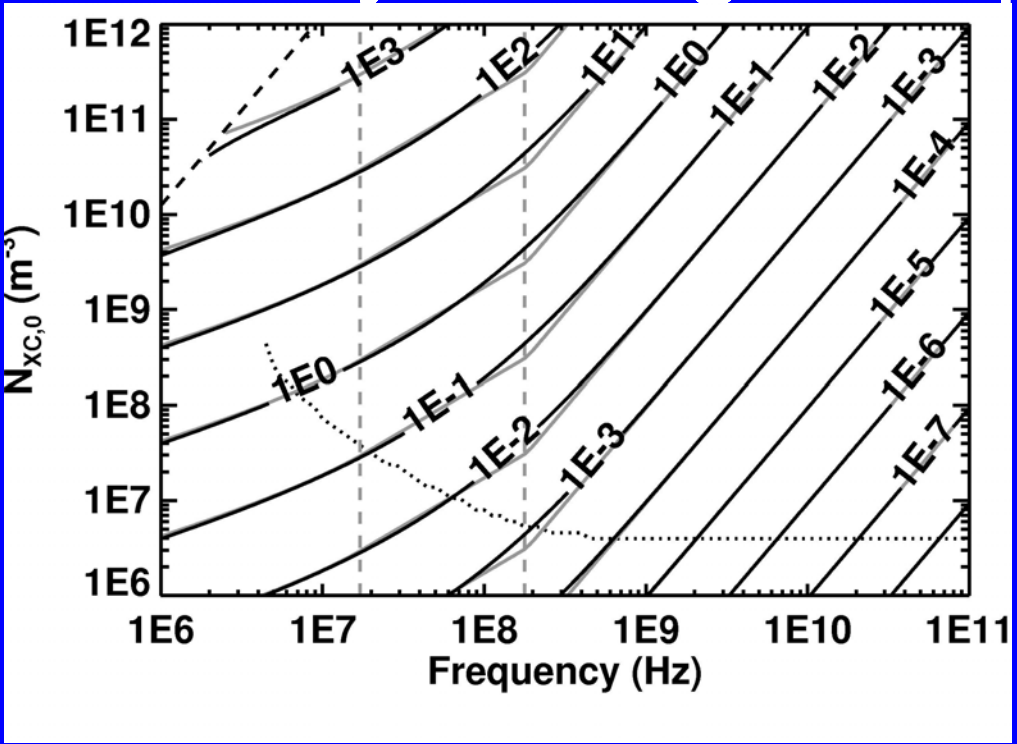
85 km meteoric layer (λ=1)



Dotted black line shows power loss from the main ionospheric layer

- Plausible N_{XC} is 2E10 m^{-3}
- Thus P > 1 dB only for f < 16 MHz
- P exceeds power loss from main ionospheric layer if N_{XC} > 3E9 m^{-3}

Predicted 35 km cosmic ray layer (λ=1, not always in high freq. limit)

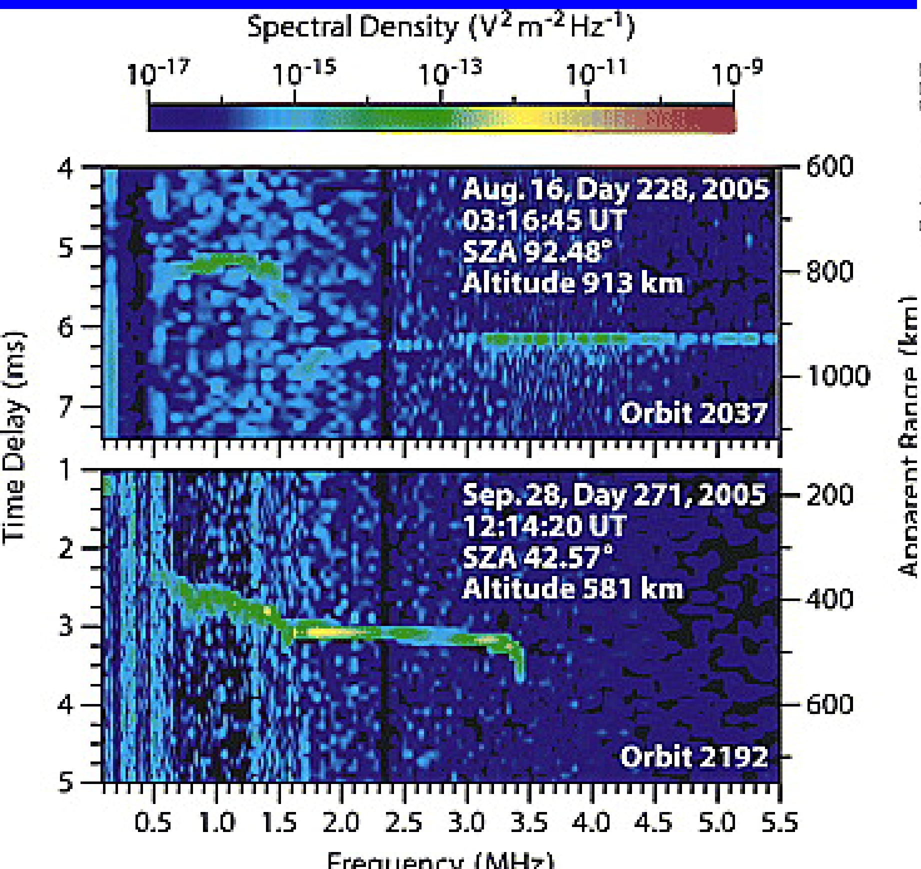


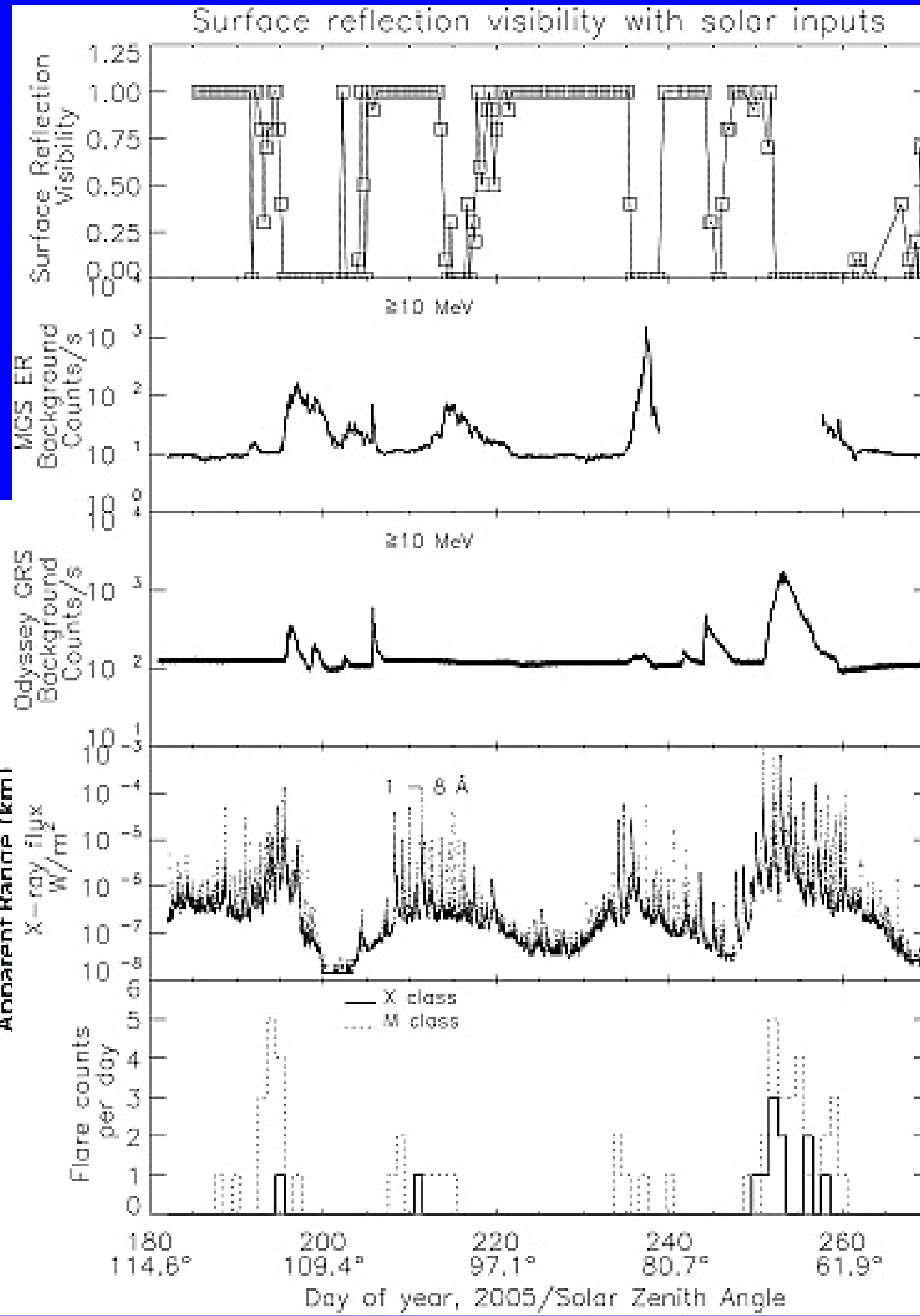
Dotted black line shows power loss from the main ionospheric layer

- Predicted N_{XC} is 1E8 m^{-3}
- Thus P > 1 dB only for f < 5 MHz
- P exceeds power loss from main ionospheric layer for f>10 MHz if N_{XC} > 1E8 m^{-3}
- This is most attenuating layer for f>50 MHz

MARSIS

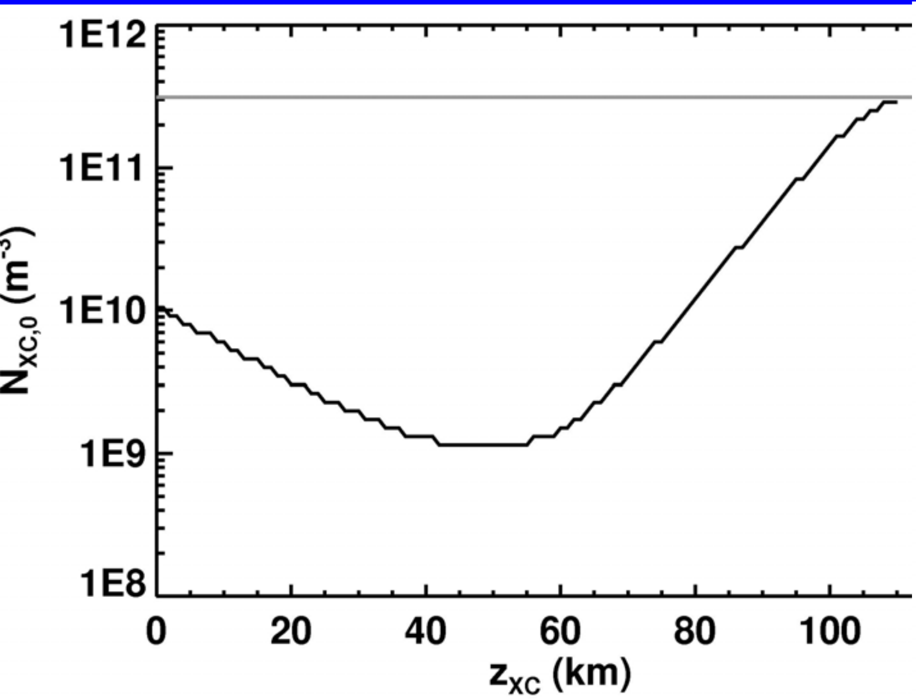
- Lack of surface reflections begin at onset of solar energetic particle events
- Endure for days after end of event

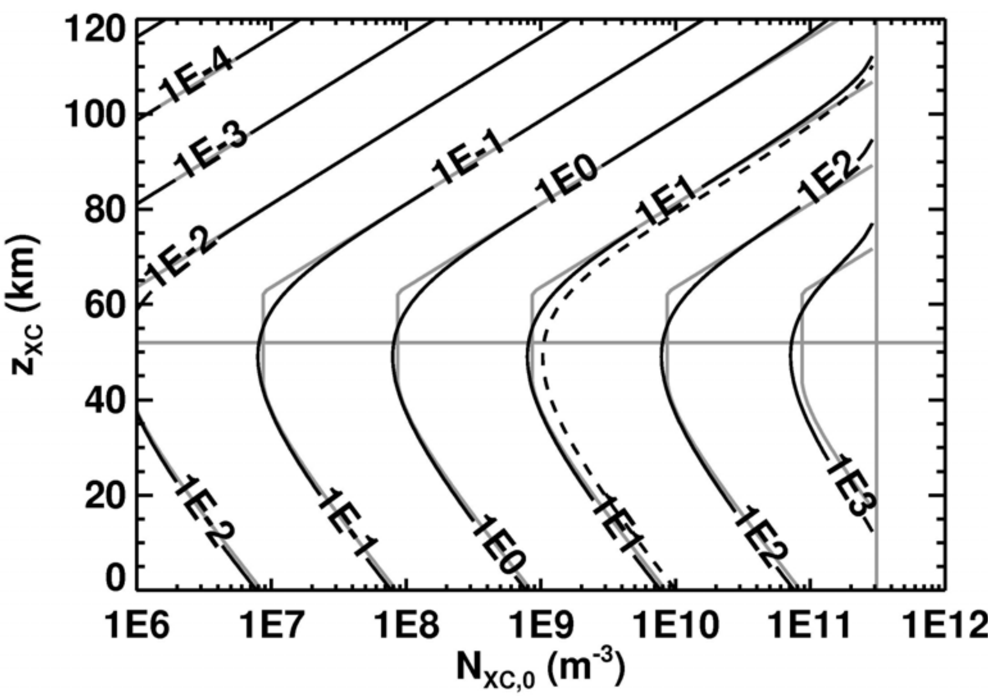




Required power loss is 13 dB

- N = 1E9 m^{-3} at optimal altitude of 50 km
- N > 1E10 m^{-3} at z < 80 km
- N > 1E11 m^{-3} at z < 100 km





Relevant models

- Leblanc et al. (2002) have a peak energy deposition rate of 3E5 eV cm^{-3} s^{-1} for an SEP event
- Inferred ion production rate is 9E3 cm^{-3}, assuming 35 eV per ion-electron pair
- Inferred ion density is 2E11 m^{-3} if ions behave like O_2^+ (…but lifetime is minutes)
- Definitely an over-estimate
- Altitude of peak energy deposition is >80 km

Paradox

- Leblanc et al. (2002) and Brain et al. (2009) say altitude of peak energy deposition is above 80 km
- Why haven't ionospheric observations seen this plasma enhancement?
  - No one has looked carefully
  - Leblanc and Brain are wrong
  - SEP events that cause MARSIS blackouts differ from those studied by Leblanc and Brain
  - Electron density not proportional to energy deposition rate due to unusual ion chemistry

Next steps

- Reproduce Brain's simple approach for getting energy deposition profile from incident energy spectrum (OK)
- Find/borrow tools to obtain ion production rates from energy deposition rate (OK)
- Find/borrow tools to obtain electron density from ion production rate (HARD)
- This will generate end-to-end pathway for range of case studies

Conclusions

- Basic relationships controlling power loss caused by ionospheric layers have been determined
- Derived expressions are useful tools for future applications
- Constraints placed on plasma densities responsible for MARSIS blackouts
- Challenges remain for simulating electron density profile during extreme solar events