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Landing spacecraft on Mars and other planets: An opportunity to apply introductory physics

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The Curiosity rover safely landed on Mars after "seven minutes of terror" passing through the Martian atmosphere. In order to land safely, Curiosity had to decelerate from speeds of several kilometers per second and reach zero speed exactly upon touching down on the surface. This was accomplished by a combination of atmospheric drag on the enclosed spacecraft during the initial hypersonic entry, deployment of a large parachute, and retrorockets. Here, we use the familiar concepts of introductory physics to explain why all three of these factors were necessary to ensure a safe landing. In particular, we analyze the initial deceleration of a spacecraft at high altitudes, its impact speed if a parachute is not used, its impact speed if a parachute is used, and the duration of its descent on a parachute, using examples from Curiosity and other missions.
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I. INTRODUCTION

The fiery entry of a spacecraft into a planetary atmosphere—whether the return of astronauts to Earth, the landing of Curiosity on Mars (Fig. 1), or the descent of Huygens to Titan's surface—attracts immense public interest. It also offers an opportunity to enliven an introductory physics class, since many fundamental aspects of the behavior of such spacecraft can readily be explained by familiar physical principles.

Here, we analyze the initial deceleration of a spacecraft at high altitudes (Sec. II), its impact speed if a parachute is not used (Sec. III), its impact speed if a parachute is used (Sec. IV), and the duration of its descent on a parachute (Sec. V). Although the concepts used in this article are commonly discussed in introductory physics classes, we note that the mathematics is at a slightly higher level.

II. INITIAL DECELERATION

Spacecraft enter atmospheres at speeds comparable to orbital speeds. In some instances, such as the return of astronauts from the space station, this means the speed of a low-altitude orbit about the planet in question. In other cases, such as the Curiosity rover of the Mars Science Laboratory mission,[1] this means the speed at aphelion of an elliptical heliocentric orbit that has its perihelion at Earth and its aphelion at the planet in question (known as a Hohmann transfer orbit). The two types of entry speeds can be calculated as a demonstration of Kepler's laws of orbital motion and Newton's law of gravity.[2] Typical values are several kilometers per second, much greater than the speed of sound in planetary atmospheres.[3]

A spacecraft entering a planetary atmosphere at supersonic speed decelerates due to atmospheric drag.[4] Momentum is transferred from the spacecraft to the atmosphere at a rate that can be estimated from the principle of conservation of momentum. The spacecraft of area A traveling at speed v sweeps through a volume $Av\Delta t$ of atmospheric gas in time Δt. The mass of this amount of air is $\rho Av\Delta t$, where ρ is the local atmospheric mass density. Assuming that this amount of air is accelerated to the same speed as the spacecraft, its momentum is $\rho Av^2\Delta t$. The momentum gained by the atmosphere per unit time, equivalent to that lost by the spacecraft per unit time, is ρAv^2. This is the aerodynamic force acting on the spacecraft, a result that is surprisingly accurate. Even with sophisticated numerical simulations, the aerodynamic force on a spacecraft is typically within a few tens of percent of ρAv^2. Neglecting the effects of gravity on the spacecraft (mass m), which is reasonable in the initial stages of atmospheric entry, gives the drag equation

$$m\frac{dv}{dt} = -\rho Av^2. \qquad (1)$$

Balancing the forces acting vertically on a small parcel of gas in the background atmosphere leads to $p(z) = p(z+\Delta z) + \rho g\Delta z$, where p is the atmospheric pressure, z is the altitude, and g is the acceleration due to gravity. This consideration of forces, which is underpinned by the principle of conservation of momentum, leads to the equation of hydrostatic equilibrium, $dp/dz = -\rho g$. For an isothermal ideal gas in which the pressure is proportional to the density, both pressure and density decrease exponentially with increasing altitude: $\rho = \rho_s \exp(-z/H)$, where ρ_s is the density at the surface and H is the atmospheric scale height; this result can be used to eliminate ρ from Eq. (1).

If the spacecraft travels at an angle ϕ from the vertical, then a pair of first-order differential equations governs changes in z and v with time:

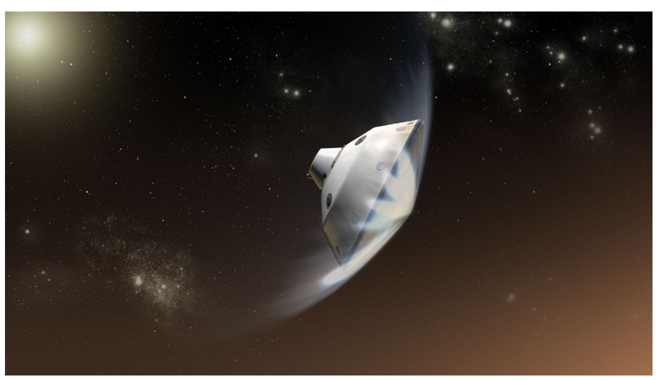

Fig. 1. Artist's conception of the entry of Curiosity into the atmosphere of Mars. NASA/JPL-Caltech image PIA 14835.

$$\frac{dv}{dt} = \frac{-\rho_s A v^2}{m} \exp(-z/H), \tag{2}$$

$$\frac{dz}{dt} = -v \cos\phi. \tag{3}$$

At the very first stages of atmospheric entry, when densities and drag forces are very small, v is effectively constant.[5] In such circumstances, the altitude satisfies

$$z = z_u - v(t - t_u)\cos\phi, \tag{4}$$

where z_u is the altitude at the top of the atmosphere when $t = t_u$. Using this result in Eq. (2) then leads to

$$\frac{dv}{dt} = \left[\frac{-\rho_s A v^2}{m} \exp(-z_u/H)\right] \exp[v(t - t_u)\cos\phi/H]. \tag{5}$$

The term in large square brackets is constant for the present assumptions, so the aerodynamic acceleration measured by an onboard accelerometer increases exponentially with time. Figure 2 illustrates that this result held for the arrival of the Pathfinder spacecraft[6] at Mars during the interval 40–80 s after atmospheric entry, while the acceleration changed by more than four orders of magnitude.[7] Determination of the characteristic timescale for changes in acceleration, $H/(v\cos\phi)$, provides a way to estimate the atmospheric scale height H. For Pathfinder, the acceleration increases by four orders of magnitude over 40 s, equivalent to a characteristic timescale of 4 s. For the entry speed of 7.4 km/s and $\phi = 75.6^\circ$, the implied scale height is 8 km, which is consistent with other observations.[8] The brevity of this characteristic timescale indicates that conditions during atmospheric entry can change very rapidly, which makes the safe passage of spacecraft through planetary atmospheres a formidable engineering challenge.

III. IMPACT SPEED WITHOUT A PARACHUTE

The intense heating that occurs during supersonic flight requires that a spacecraft be protected by a heat shield. Let us consider whether deceleration due to drag on the enclosed

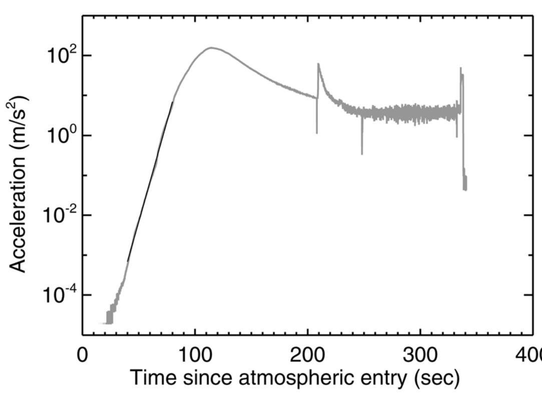

Fig. 2. Deceleration as a function of time since atmospheric entry (gray line) during the arrival of Pathfinder at Mars in 1997. The black line shows an exponential fit to a portion of the gray curve. This fit, which has a characteristic timescale of 4.3 s, illustrates that Eq. (5) holds well for this mission. In the data, a maximum deceleration of 155 m/s^2 occurs at 114 seconds, parachute deployment occurs at 209 s, and first impact occurs at 336 s. (Data source: NASA Planetary Data System dataset MPAM_0001.)

spacecraft is sufficient to ensure a safe landing speed upon reaching the surface. We again neglect gravity. Combining Eqs. (2) and (3), we have

$$\frac{dv}{dz} = \frac{\rho_s A v}{m\cos\phi} \exp(-z/H), \tag{6}$$

and after integration this becomes

$$\ln(v/v_u) = \frac{-[\rho(z) - \rho_u]HA}{m\cos\phi}. \tag{7}$$

Here ρ_u is the density at the top of the atmosphere, where the corresponding speed is v_u. Because the location of "the top" of the atmosphere is arbitrary we can make ρ_u arbitrarily small, which leads to

$$\ln(v/v_u) = \frac{-\rho(z)HA}{m\cos\phi} = \frac{-p(z)A}{mg\cos\phi}. \tag{8}$$

Because atmospheric pressures and planetary gravities are more commonly used and tabulated than are densities and scale heights, here we have replaced the atmospheric density by the pressure p using $p = \rho g H$. Note that $M_A \equiv \rho_s HA = p_s A/g$ is the mass of a vertical atmospheric column of area A, and $M_A/\cos\phi$ is the total mass of the atmospheric gas swept up by the spacecraft.

For the Curiosity landing, $p_s = 10^3$ Pa, $g = 3.7\,\text{m/s}^2$, $A = 16\,\text{m}^2$, $m = 2400$ kg, and $v_u = 6$ km/s. A vertical entry ($\phi = 0$) implies $v = 0.2 v_u = 1.2$ km/s, while an angled entry with $\phi = 60^\circ$ implies a much slower speed, despite $\cos\phi$ only changing from 1 to 0.5. This angled entry implies $v = 0.03 v_u = 180$ m/s, which is still 400 mph, far in excess of any plausible safe landing speed. Since the planet is round, the entry angle ϕ cannot be increased all the way to 90°; at some point, the descending trajectory will become merely a grazing trajectory and the spacecraft will skip out of the atmosphere and return to space.[9] Thus, it is clear that additional means of deceleration are required to ensure a safe landing speed for Curiosity, as described in Sec. IV.

Figure 3 uses Eq. (8) to show how speed depends on altitude for a range of possible values of $M_A/(m\cos\phi)$. As a spacecraft descends its speed barely changes until $M_A/\cos\phi > 0.1$, and once $M_A/\cos\phi > 1$ the speed decreases precipitously. Equation (8) can also be used to explore the sensitivity of impact speed to surface pressure and to spacecraft mass and area, properties over which the mission designer has at least some control. Due to the functional form of this equation, the impact speed is highly sensitive to the independent variables—one of the reasons why Mars landers are targeted to low altitude, high atmospheric pressure regions. The neglect of gravity may be an unrealistic approximation, but including the gravitational attraction of the planet will increase, not decrease, the impact speed from that estimated by Eq. (8). Gravitational acceleration also causes the angle ϕ, previously constant, to decrease as the trajectory becomes more vertical.

IV. IMPACT SPEED WITH A PARACHUTE

We have seen that the deceleration provided by a heat shield alone may be insufficient to ensure a safe landing speed. In this case, the most common solution is to increase the effective area of the spacecraft by deploying a parachute.

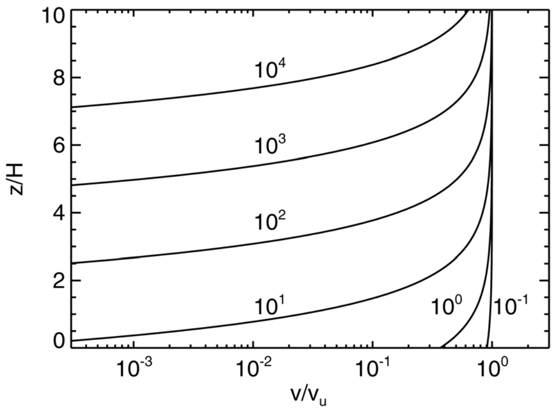

Fig. 3. Prior to parachute deployment or retrorocket ignition, the speed of a spacecraft of mass m depends on the mass of atmospheric gas that is swept up by the spacecraft. The six solid lines show how the ratio of speed v to initial speed v_u depends on altitude z (in units of scale height H) for different values of $M_A/m\cos\phi$. Here M_A is the mass of a vertical atmospheric column above altitude $z=0$ that has an area equal to the spacecraft area, and ϕ is the angle between the spacecraft velocity and the vertical, so that $M_A/\cos\phi$ is the mass of atmospheric gas swept up by the spacecraft by altitude $z=0$.

Deployment at as high an altitude as possible is desirable to minimize the landing speed, but parachutes cannot be safely deployed until the spacecraft's speed is close to the speed of sound, roughly $\sqrt{kT/\mu}$, where k is Boltzmann's constant, T is temperature, and μ is the mean molecular mass of atmospheric gas. The phrase "close to" should be interpreted broadly: Curiosity's parachutes were designed to open at Mach 2. At the exceptionally high hypersonic speeds characteristic of the spacecraft's first encounter with the fluid atmosphere, a parachute would be torn apart during deployment. Inserting the speed of sound, typically a few hundred meters per second, into Eq. (8) provides a way to estimate the atmospheric conditions at parachute deployment.

During descent on a parachute gravity can no longer be neglected. Once a parachute is deployed the spacecraft motion soon becomes vertical and the equation of motion is

$$m\frac{dv}{dt} = mg - \rho A_p v^2, \tag{9}$$

where A_p is the area of the parachute. Because the effective area of the spacecraft has suddenly increased substantially, the speed abruptly decreases at parachute deployment, eventually reaching terminal velocity v_T, when gravitational and drag forces are equal ($mg = \rho A_p v_T^2$). Although the spacecraft remains at terminal velocity as it descends on the parachute, its speed will decrease due to the increase in atmospheric density. The terminal velocity at the surface satisfies

$$v_T^2 = \frac{mg^2H}{p_s A_p}. \tag{10}$$

Again we have replaced density using $p = \rho g H$. For Curiosity, the mass during parachute descent was about half of the entry mass due to the ejection of the heat shield after parachute deployment. The descent of Curiosity on its 200 m^2 parachute was imaged by the Mars Reconnaissance Orbiter from its orbit around Mars (Fig. 4).[10] Despite this large parachute, Eq. (10) predicts that terminal velocity at the surface would be 30 m/s or 70 mph, which would result in a high speed crash on the surface. Although this is a significant improvement over the

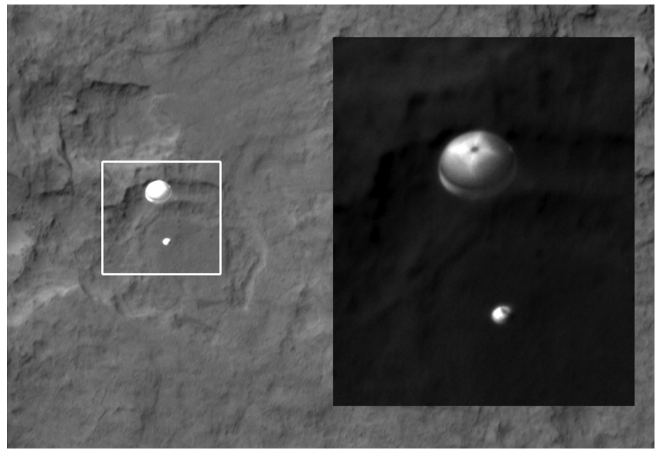

Fig. 4. A view from Mars orbit of Curiosity descending on its parachute. NASA/JPL/University of Arizona ESP_028256_9022.

hundreds of meters per second predicted in the previous section, a further means of deceleration is necessary; this is why Curiosity's landing system also included retrorockets that operated during its final approach to the surface.

The dangers of parachute failure were illustrated by the Genesis mission, which returned solar wind samples to Earth in 2004;[11,12] its parachute failed to open and it landed at nearly 90 m/s (200 mph), resulting in the unfortunate situation shown in Fig. 5. Fortunately, the precious samples were not completely ruined by contamination.

Terminal retrorockets require fuel. If deceleration from terminal velocity to zero using chemical propulsion requires a large mass of fuel, where "large" can be considered to be a fuel mass that approaches the total spacecraft mass, then a meaningful mass of scientific payload cannot be delivered safely to the surface. The impulse provided by a system of retrorockets is $m_{\rm fuel}I_{sp}$, where $m_{\rm fuel}$ is the mass of fuel consumed and I_{sp} is the specific impulse of the retrorockets, a quantity that is related to the chemical energy density of the fuel and the design of the retrorocket system. For complete deceleration, this impulse must equal the momentum of the spacecraft mv_T. Because the specific impulse of the hydrazine rockets used by Curiosity was about 2000 m/s, fifty times the calculated terminal velocity of Curiosity on its parachute, the required fuel mass is only one-fiftieth of the spacecraft mass.[13] It was therefore feasible for Curiosity to decelerate to

Fig. 5. Closeup view of the damaged Genesis capsule after its return to Earth. NASA/JPL-Caltech.

a near-zero safe landing speed using retrorockets and still land a substantial scientific payload on the surface of Mars.

V. LONG-DURATION PARACHUTE DESCENT

In dense atmospheres such as those of Venus and Titan, terminal velocity is attained high above the surface. In these instances, the interval between parachute deployment and landing can be extremely long. At Titan, the Huygens probe decelerated from supersonic speeds using an 8.31-m diameter main parachute, but then discarded it and deployed a smaller 3.03-m diameter stabilizer parachute in order to reach the surface before its batteries ran out.[14,15]

For the simpler case of a single parachute, the time interval between parachute deployment and landing can be calculated as follows. From Eqs. (9) and (10), we have

$$\left(\frac{dz}{dt}\right)^2 = \frac{mg}{A_p \rho(z)} = \frac{mg}{A_p \rho_p} \exp[(z - z_p)/H], \tag{11}$$

where parachute deployment occurs at an altitude z_p and atmospheric density ρ_p. This gives

$$\begin{aligned} \frac{dz}{dt} &= -\sqrt{\frac{mg}{A_p \rho_p}} \exp[(z - z_p)/2H] \\ &= -v_p \exp[(z - z_p)/2H], \end{aligned} \tag{12}$$

where v_p is the terminal velocity of the spacecraft beneath the parachute at altitude z_p. Integration of this equation yields

$$\exp[-(z - z_p)/2H] = 1 + \frac{v_p(t - t_p)}{2H}, \tag{13}$$

where $t = t_p$ at parachute deployment. The descent speed is then given by

$$\frac{dz}{dt} = \frac{-v_p}{1 + v_p(t - t_p)/2H}. \tag{14}$$

For $t - t_p \gg 2H/v_p$, the descent speed reduces to

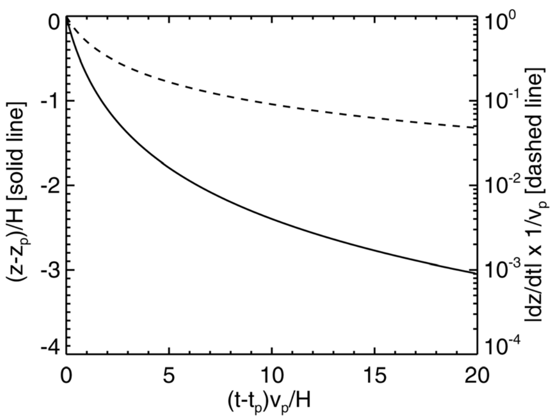

Fig. 6. The altitude z and speed $|dz/dt|$ of a spacecraft descending on a parachute at terminal velocity are related to the atmospheric scale height H, the terminal velocity at parachute deployment v_p, and the time since parachute deployment. Here z_p is the altitude of parachute deployment so $z - z_p$ is the change in altitude since parachute deployment; similarly, t_p is the time of parachute deployment so $t - t_p$ is the time since parachute deployment. We plot dimensionless altitude [$(z - z_p)/H$, solid line] and dimensionless velocity ($|dz/dt| \times 1/v_p$, dashed line) as functions of dimensionless time since parachute deployment [$(t - t_p)v_p/H$].

Table I. Information on several planetary atmospheres. Values are at the 10^5 Pa pressure level for Jupiter and the surface for all other objects. Scale heights are calculated using $p = \rho g H$. (Data from Ref. 16)

Object	Temperature (K)	Pressure (kPa)	Density (kg/m^3)	Gravity (m/s^2)	Scale height (km)
Venus	735	9000	65	9	16
Earth	288	100	1	10	8
Mars	214	0.6	0.02	4	11
Jupiter	165	100	0.2	23	25
Titan	94	150	6	1	20

$$\frac{dz}{dt} = \frac{-2H}{t - t_p}. \tag{15}$$

Figure 6 shows how altitude and descent speed depend on time. For Huygens at Titan, parachute deployment occurred at 157 km altitude and an atmospheric pressure of 200 Pa. These conditions correspond to $v_p = 33$ m/s for the large main parachute. If the small stabilizer chute had been deployed immediately, rather than 15 min later, then v_p would have been a significantly faster 90 m/s. If these two values of v_p are used in Eq. (13), then descent intervals of 17 h and 6 h, respectively, are predicted, which illustrates the effect of parachute size on descent duration. Since Huygens did not decelerate to terminal velocity immediately upon parachute deployment, these values are over-estimates (Huygens actually took 2.5 h to reach the surface).

VI. SUMMARY

We have used the familiar concepts of introductory physics to analyze the initial deceleration of a spacecraft at high altitudes, its impact speed if a parachute is not used, its impact speed if a parachute is used, and the duration of its descent on a parachute. Upon atmospheric entry, deceleration increases exponentially with time and the characteristic timescale for its increase can be used to determine the atmospheric scale height. Prior to parachute deployment, the ratio of the spacecraft speed to the entry speed is solely determined by the ratio of atmospheric mass swept up by the spacecraft to the spacecraft mass. Once the swept-up mass equals the spacecraft mass, deceleration is very rapid. After parachute deployment, which drastically increases the area of the spacecraft, the spacecraft speed approaches terminal velocity in the ambient atmosphere. The terminal velocity

Table II. Information on several spacecraft. (Data from Ref. 17 and sources therein.)

Spacecraft	Entry mass m (kg)	Heatshield diameter d (m)	Entry speed v_u (km/s)	Entry angle ϕ (deg)	Parachute diameter d_p (m)
Pioneer Venus Large probe	302	1.4	11.7	56	4.94
Genesis	206	1.5	11.0	82	N/A
Curiosity	3260	4.5	5.5	75	19.7
Galileo	339	1.3	47.4	82	3.8
Huygens	320	2.7	6.0	24	8.31/3.03

decreases as the spacecraft descends deeper into the atmosphere and the spacecraft speed is inversely proportional to the time elapsed since parachute deployment.

Table I provides information on conditions at Venus, Earth, Mars, Jupiter, and Titan. Table II provides information on selected spacecraft that have flown to those planetary bodies. This information can be used in conjunction with the relationships derived in this work to investigate how these spacecraft navigated their journeys through the atmospheres of these planets.

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Coulometer

Thomas Edison's first commercial electrical power distribution system was set up in 1882 in lower Manhattan. This was a direct current system with large, copper conductors leading from the power station, and smaller wires branching out to the individual users. Initially, the coulometer system was used to calculate the power used by each subscriber. The key element is the zinc electroplating system, with electrodes of zinc suspended in a bath of zinc sulphate. At regular intervals the cathode was weighed to give the amount of zinc deposited on it. A straightforward calculation, based on the fact that 96,500 coulombs are needed to deposit 65.4 g of zinc, gave the total charge delivered to the consumer. The energy consumption in Watt-seconds was obtained by multiplying this charge by the voltage across the user's system. (Notes and photograph by Thomas B. Greenslade, Jr., Kenyon College)