ANGLE OF REPOSE-LIMITED SHAPES OF ASTEROIDS. Paul Withers, *Lunar and Planetary Laboratory, University of Arizona, Tucson AZ 85721, USA, (withers@lpl.arizona.edu).*

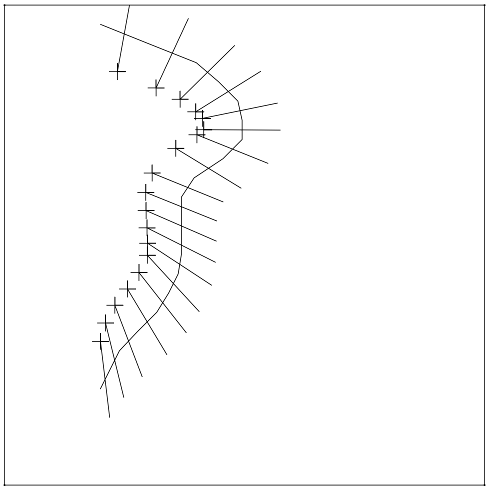

Figure 1: Local gravity vectors on an irregularly shaped body. Length of arrow is proportional to strength of local gravity, crossed end of arrow indicates direction of gravity.

1 Introduction

The existence of giant craters on many asteroids, low asteroidal densities, and low asteroidal rotation rates suggest that asteroids larger than a few hundred metres are porous, nearly strengthless "rubble piles" composed of monolithic subunits with sizes on the order of tens of metres [Asphaug, 1999; Ostro et al., 1999].

A "rubble pile" asteroid may be nearly strengthless, but it can still support topography via frictional forces, just like a heap of sand supports itself. If this mechanism does control asteroidal shapes, then no slopes greater than 30^{o}, a typical angle of repose, should be seen on asteroids. Asteroid flyby imaging has revealed only a few slopes steeper than 30^{o}. A small number of steep slopes can be explained away as occuring on the boundaries between monolithic subunits and do not invalidate this mechanism.

Observed asteroidal shapes are not angle of repose-limited, they contain regions of shallower slope. What would an angle of repose-limited shape look like? Such a shape would represent an end-member for possible asteroidal shapes, being as far removed from a sphere as possible. As such it is interesting to try to find such a shape and investigate its properties.

2 Results

I generated lots of elliptical and irregular shapes then calculated their surface slopes, hoping that some shapes would approach being angle of repose-limited. I tried to find shapes with mean slopes as large as possible and maximum slopes not exceeding 30^{o}. For the irregular shapes, a few slopes were allowed to exceed 30^{o}, as has been seen on real asteroids. When calculating mean slopes in this case, the few slopes greater than 30^{o} were neglected. The irregular shapes proved surprisingly unsuccessful. Of 200 irregular shapes, only 5 had mean slopes greater than 15^{o}, and none of those had mean slopes greater than 18^{o}. One of the five most successful shapes is shown in Figure 1.

The elliptical shapes were more successful. A shape with axial ratio $\sim$ 0.3 has a mean slope $\sim 20^{o}$.

An elliptical shape is completely defined by one parameter, its axial ratio, and hence its maximum and mean slopes are both simply functions of this single parameter. It should be possible to find analytical expressions for both these slopes in terms of this parameter. At the time of writing, I have not attemped this. If analytical expressions could be found and extended to the case of rotating and/or triaxial ellipsoids then simple shapes which are close to angle of repose-limited can be studied in detail and compared to observed asteroidal shapes.

3 Rotation

For a range of physically realistic rotation rates and densities, there was always an elliptical shape with a mean slope $\sim 20^{o}$ and maximum slope less than 30^{o}. As rotation rate increased, the axial ratio of the "best" ellipse changed but did not change monotonically. Again, an analytical description would aid insight here.

The "best" irregular shapes continued to have mean slopes $\sim 15^{o}$ as the effects of rotation were increased. Unlike the elliptical shapes, "good" irregular shapes tended to stay "good" as the effects of rotation were increased.

4 Conclusions

There are sound reasons for wondering what an angle of repose-limited asteroidal shape might look like. 200 axisymmetric shapes, assumed to be homogeneous, generated by a restricted random walk approach yielded only a handful of examples with mean slopes greater than 15^{o} when shapes with slopes significantly exceeding 30^{o}, a typical angle of repose, were excluded. An axisymmetric elliptical shape, assumed to be homogeneous, with axial ratio $\sim$ 0.3 had a mean slope of 20^{o} and no slopes exceeding 30^{o}. When physically realistic rotational effects were included, similar results were obtained for elliptical and irregular shapes, though with a different elliptical shape being closest to angle of repose-limited.

5 Acknowledgements

Jay Melosh

6 References

Asphaug, E. 1999, Survival of the weakest, *Nature*, **402**, 127–128

Ostro, S. J., and 19 co-workers 1999, Radar and Optical Observations of Asteroid 1998 KY26, *Science*, **285**, 557–559